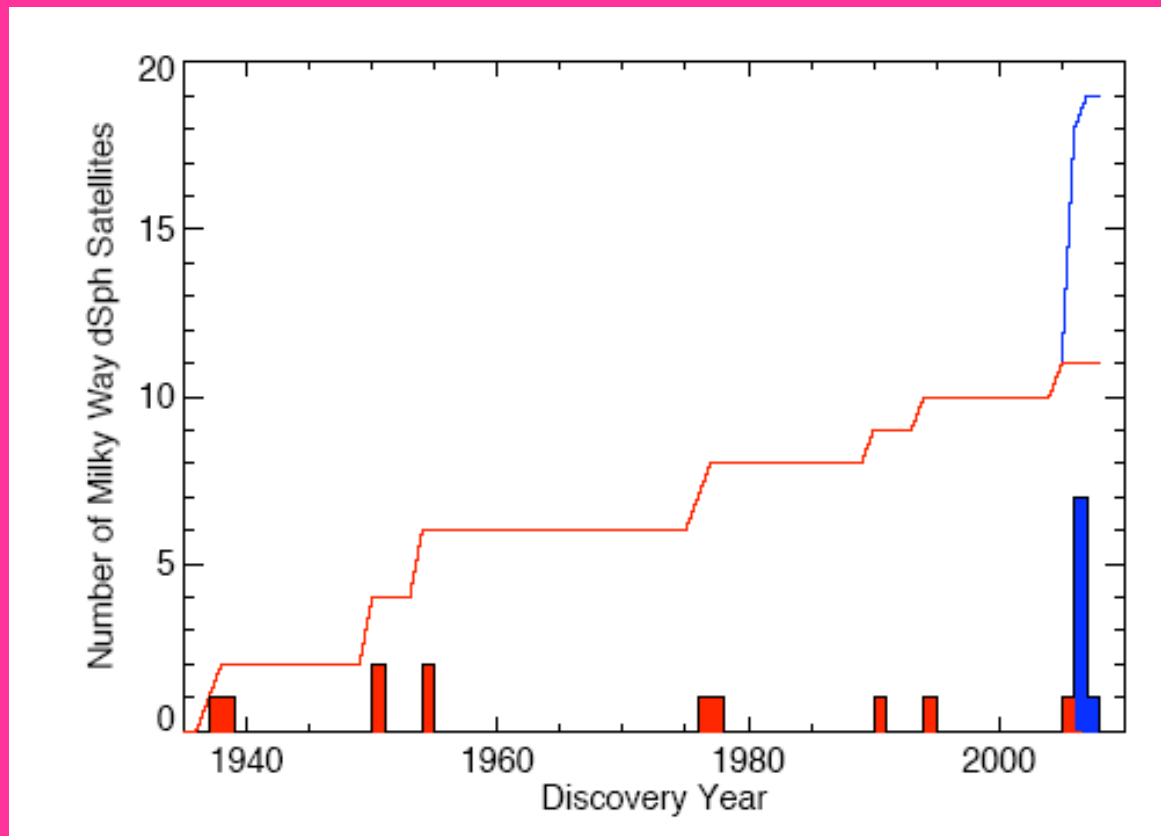
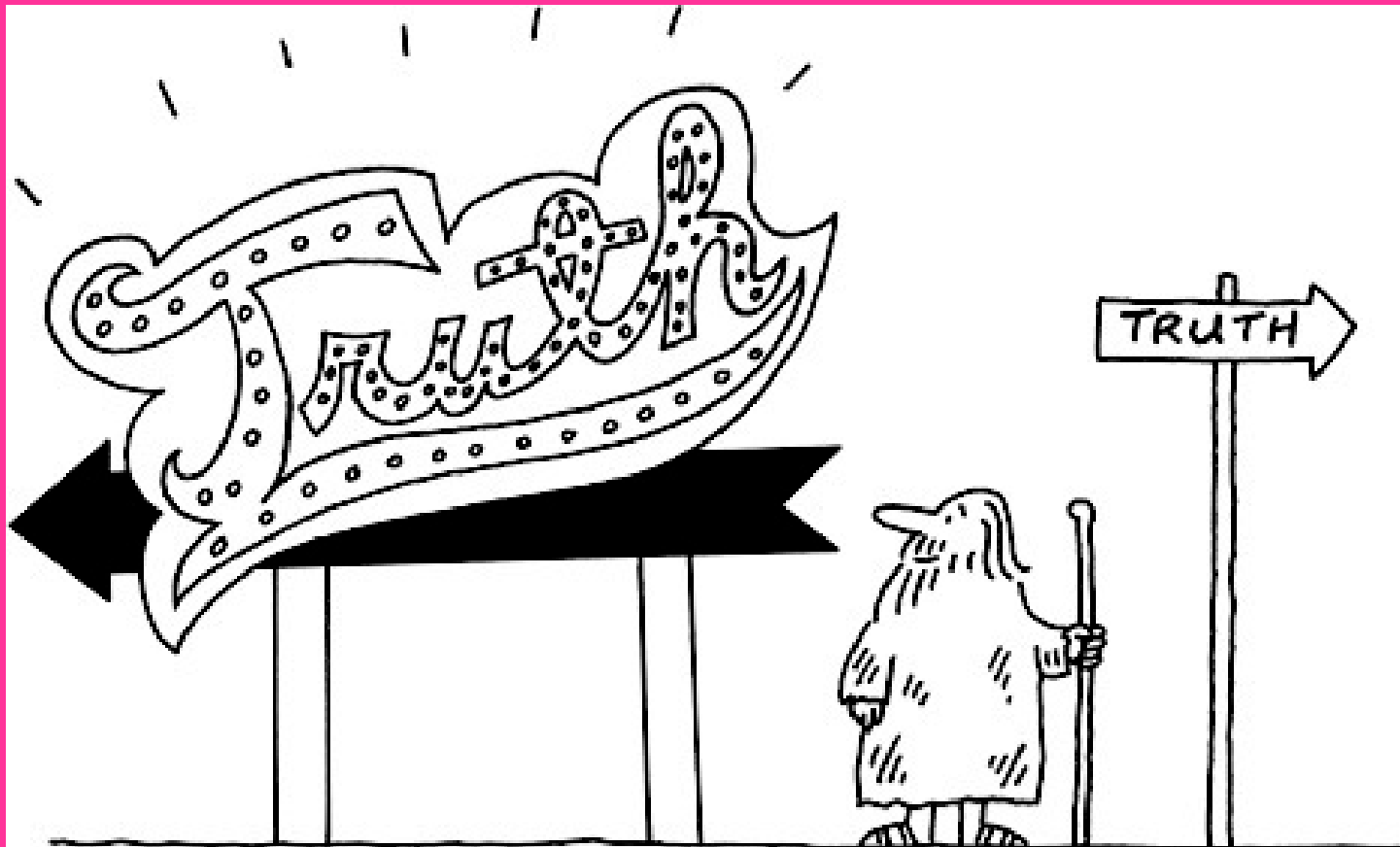


Dark matter in dSph galaxies?

Gerry Gilmore
IoA Cambridge

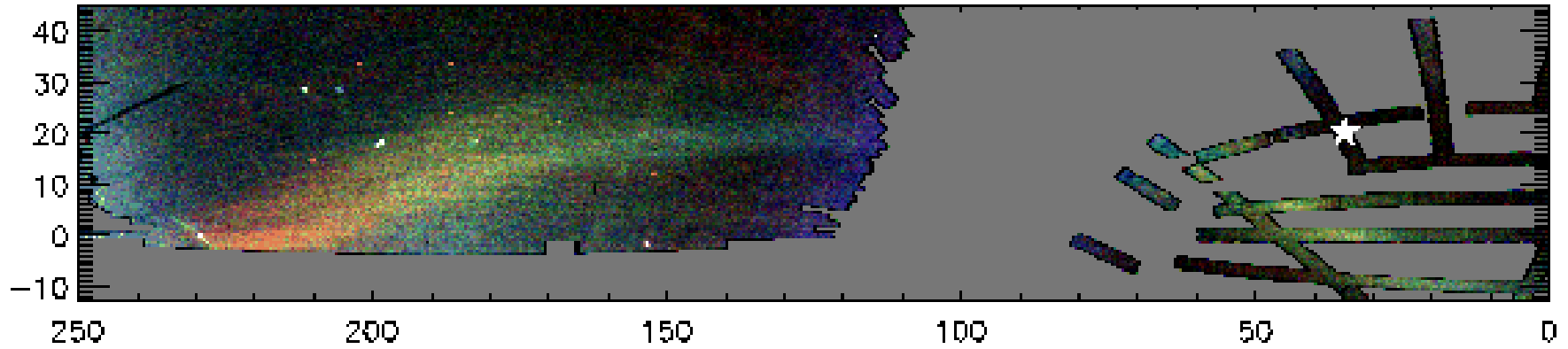


Cambridge
blue



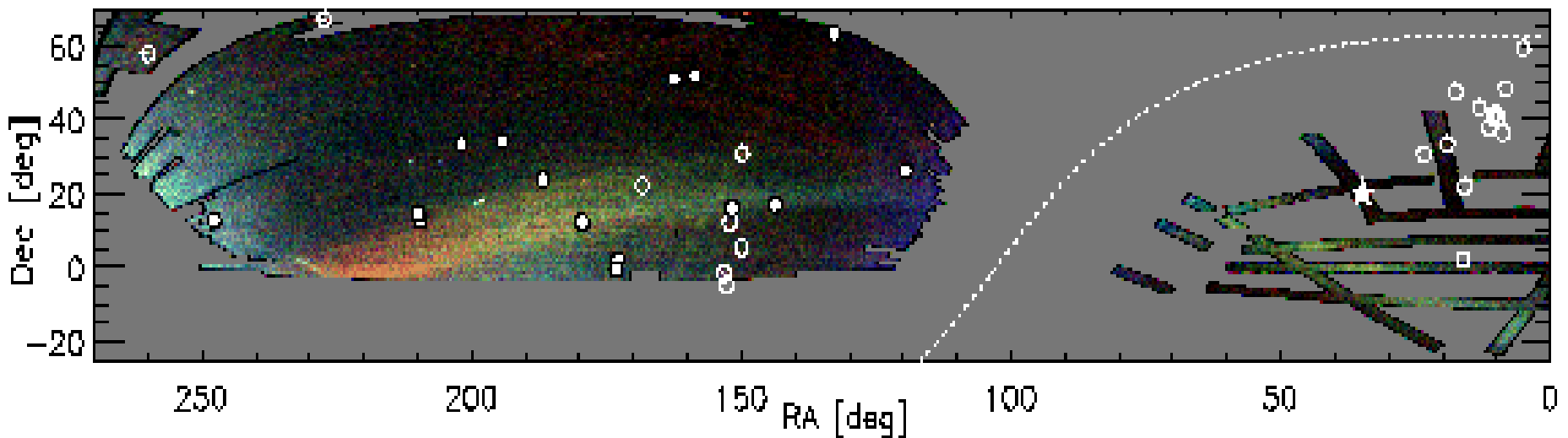
CParvathi

Field of Streams - updated



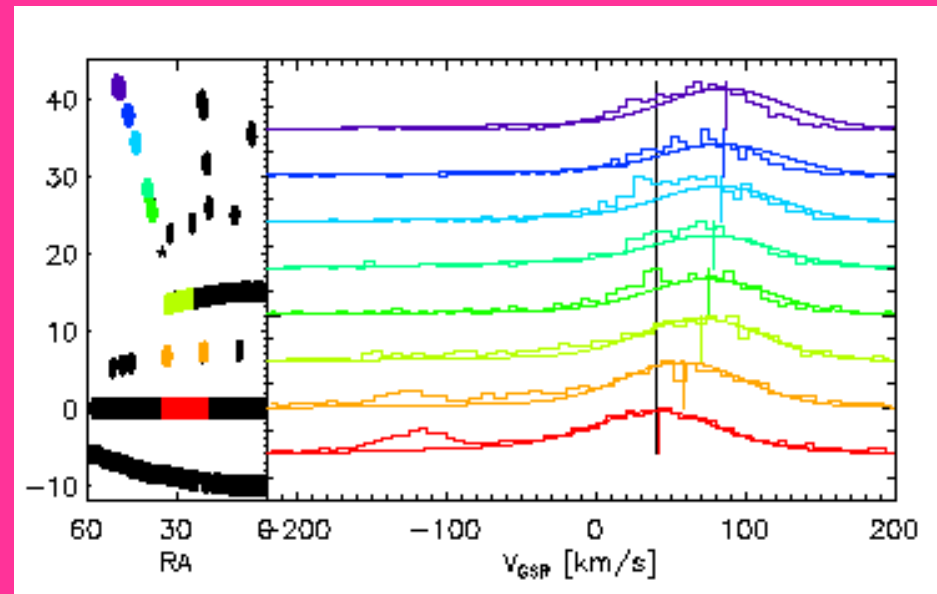
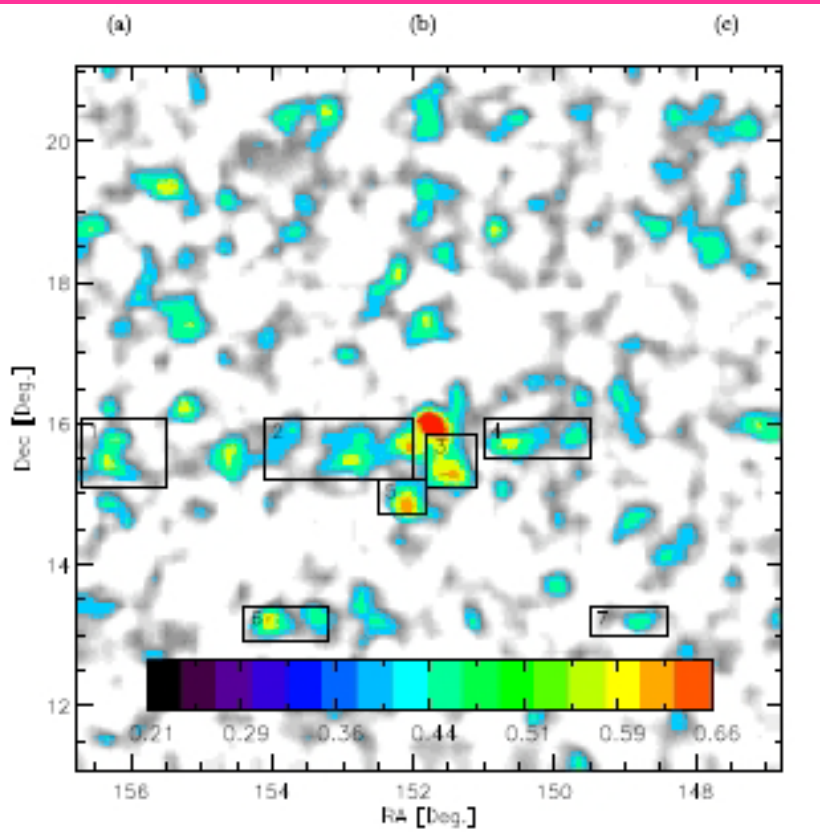
Segue-2 discovery paper : Belokurov et al arXiv: 0903.0818

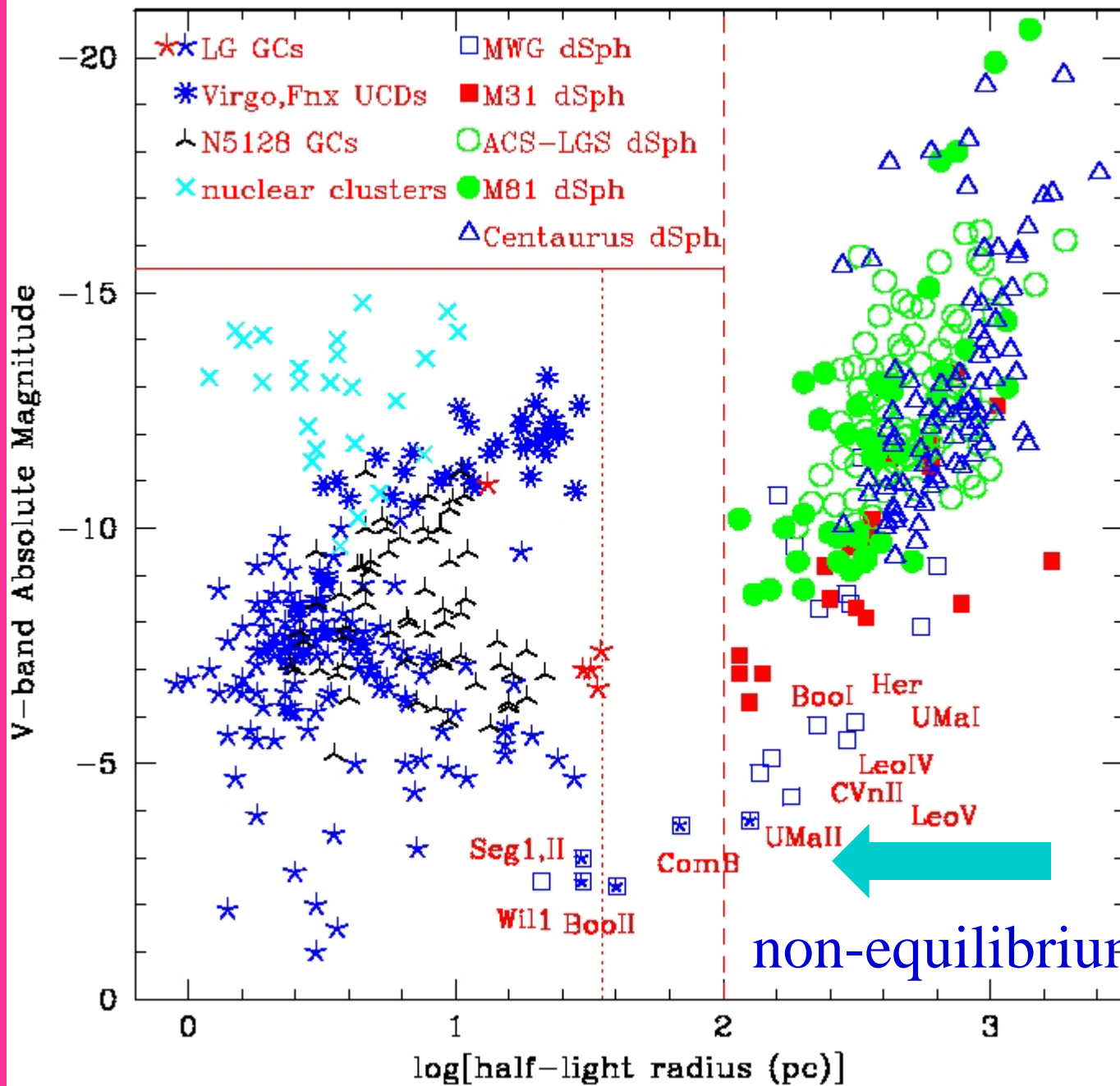
Classical dSph – open circles; ultra-faint – closed
note the several along the Sgr tail



Segue-1 is embedded in Sgr tails, in a complex velocity field (Belokurov et al) → ultra-faints are complex, nonequilibrium?
 Subaru deep imaging – Okamura talk – the local faint ones
 Are extended and messy...
 most `sizes` are significantly under-estimated

Seg-II– another
 halo stream connection





From kinematics to dynamics: Jeans equation

- Relates spatial distribution of stars and their velocity dispersion tensor to underlying mass profile

$$M(r) = -\frac{r^2}{G} \left(\frac{1}{\nu} \frac{d\nu\sigma_r^2}{dr} + 2 \frac{\beta\sigma_r^2}{r} \right)$$

- ◆ (i) identify least model-dependent enclosed mass, and compress kinematic and luminosity data to a single number – an enclosed mass.
- ◆ (ii) determine mass profile from projected dispersion profile, with assumed isotropy, and smooth functional fit to the light profile
- ◆ (iii) assume a parameterised mass model $M(r)$ and velocity dispersion anisotropy $\beta(r)$ and fit dispersion profile to find best forms of these (for fixed light profile)
- ◆ Or (iv) do it properly, with the DF, not moments

$M < r$

- Illingworth
1976
-
- Mateo 1990s
- Strigari,
Walker,
Mamon,
Wolf...

 $M = L?$

- Mateo et al
1990s
- Wilkinson et al
2002
- Koch et al
- Lokas
- Many more

 $M(r)$

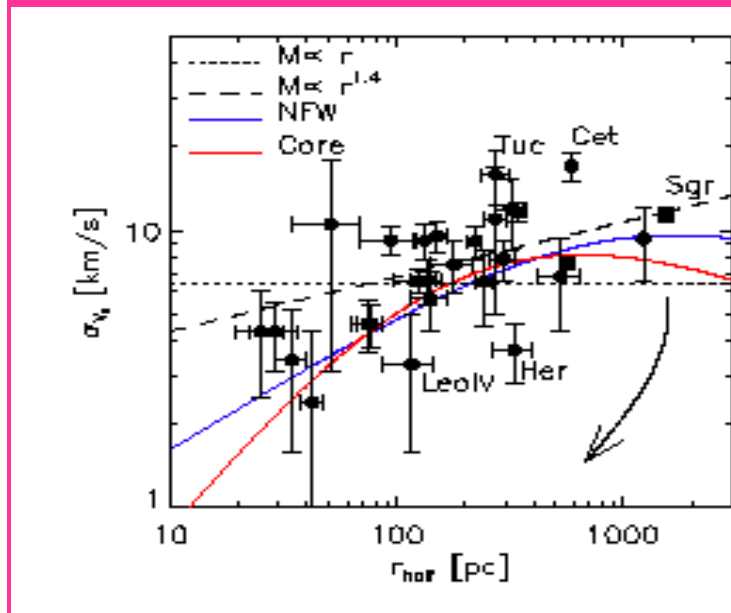
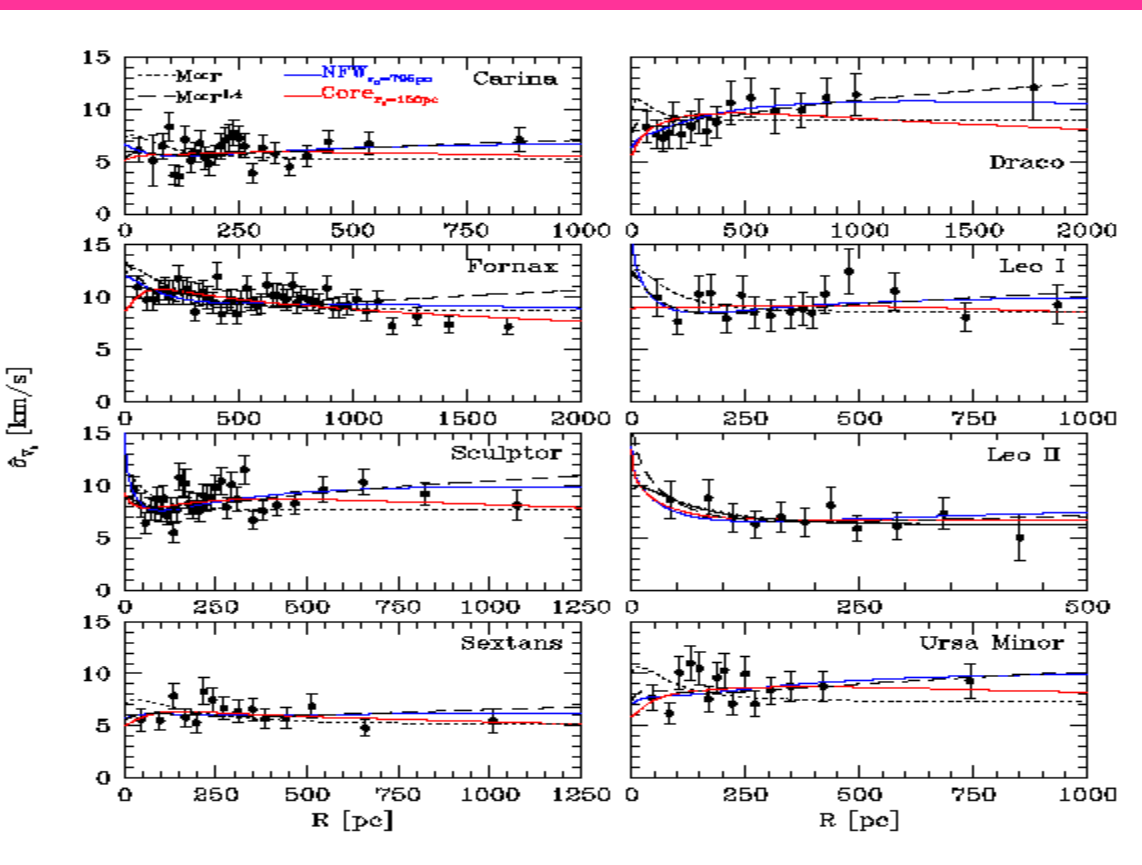
- MB, BE, FD, RJ....
- Eddington, Jeans,
Fricke, Chandrasekhar,
Miyamoto, Nagai,
Toomre, Lynden-Bell,
Dehnen, deZeeuw,
Evans, Kent & Gunn,
Merrifield & Kent,
Kuijken & Gilmore,
Wilkinson & KEG,
Wu & Tremaine,
Lokas.....

Hundreds of others

Plus proxy methods based on internal abundance dispersion

Lots of lovely kinematics, and correlation – dispersion vs size

$M < r_{\text{metric}}$: dispersion roughly const with radius, value roughly correlated with size \rightarrow reduce to single parameter:
 analogous to $M = \text{cst.} \cdot v^2 \cdot r$



Binaries quantified
 - for giants
 Precision tolerably
 well known

THE DYNAMICS OF RICH CLUSTERS OF GALAXIES. I. THE COMA CLUSTER

S. M. KENT

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138

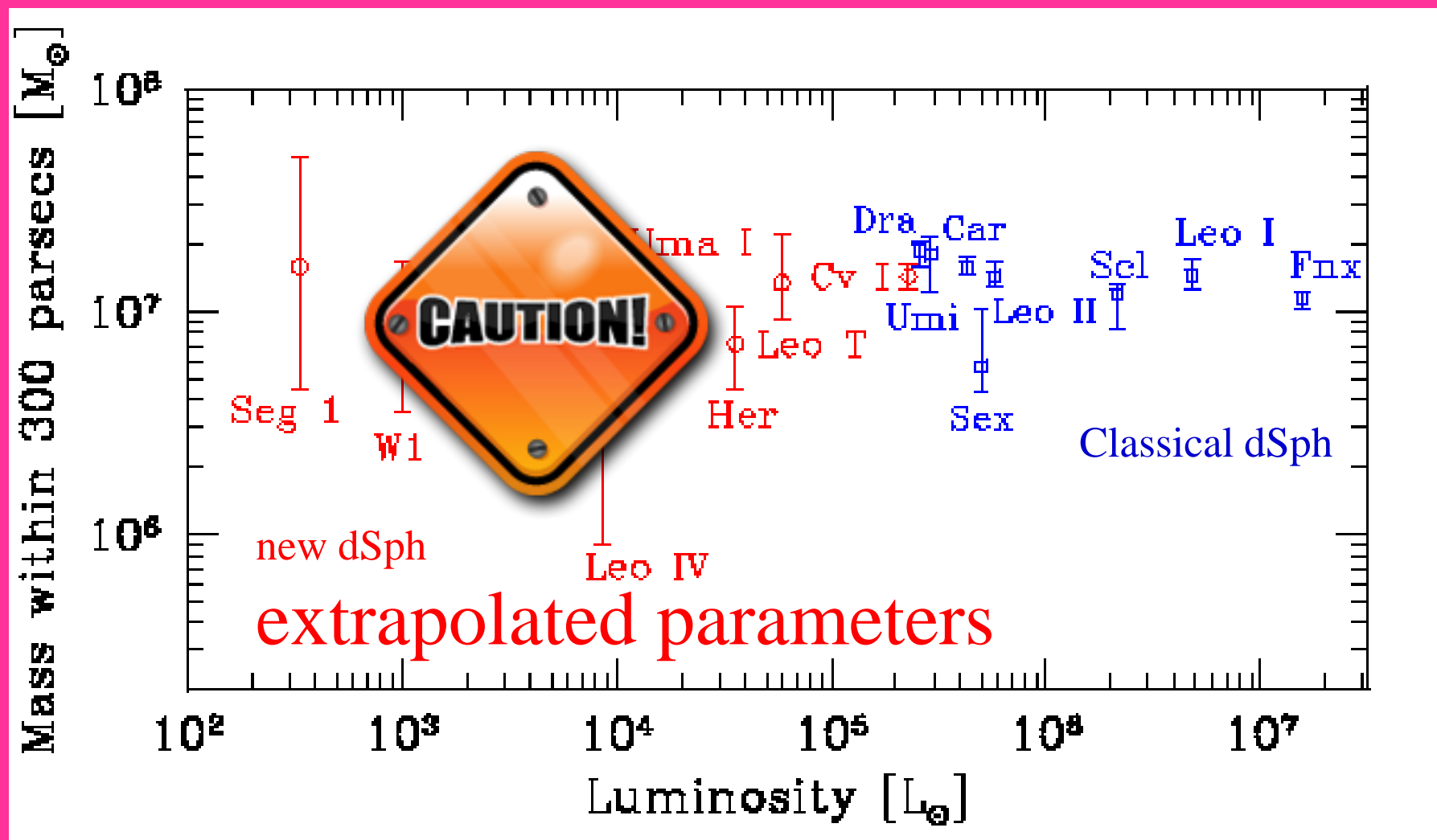
J. E. GUNN

The structure and dynamics of the Coma cluster are analyzed using self-consistent equilibrium dynamical models. Observational material for Coma is culled from a variety of sources. Projected surface, density, and velocity-dispersion profiles are derived extending out to a radius of 3° from the cluster center, which are essentially free from field contamination. Segregation of galaxies by luminosity and morphology are discussed and a quantitative estimate of the latter is made. The method of constructing self-consistent dynamical models is discussed. Four different forms of the distribution function are analyzed allowing for different possible dependences of f on energy and angular momentum. Properties of typical models that might resemble actual clusters are presented, and the importance of having velocity-dispersion information is emphasized. The effect of a central massive object such as a cD galaxy on the core structure is

The total mass in Coma as presented by our models turns out to be quite insensitive to the specific model used. We find the total projected mass inside a radius of 3° to be $2.9 \times 10^{15} h_{50}^{-1} M_\odot$. For the moment, we do not attempt to extrapolate beyond this radius.

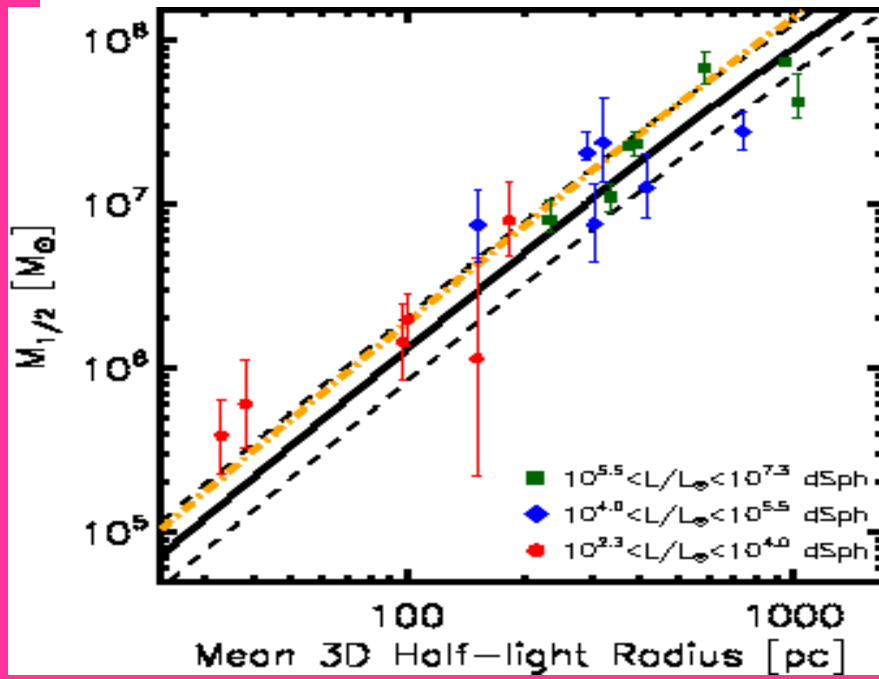
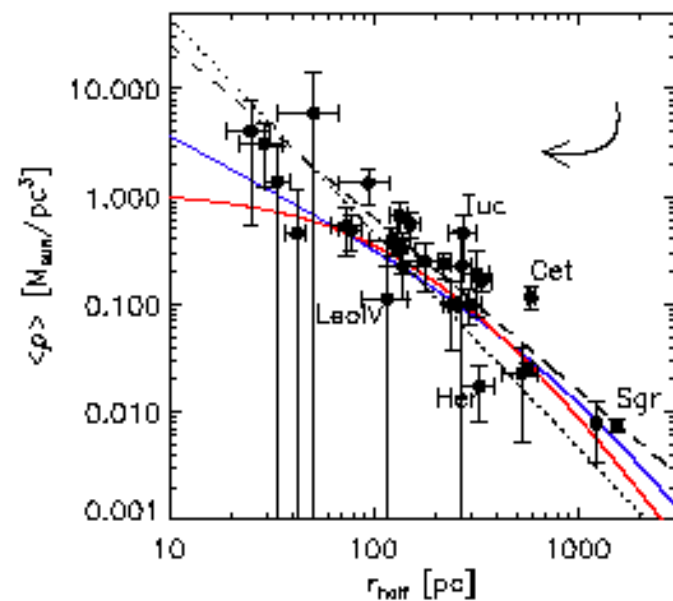
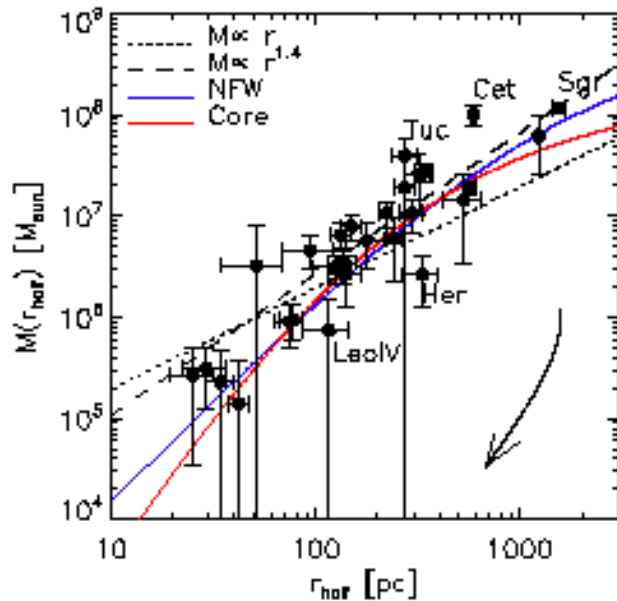
Mass within half-light radius long known to be robust: why?

- Compress kinematics to an enclosed mass in a metric size:
 Concept valid only when constrained by data
 – M300 mixes data and model. Better is an object-specific scale



Strigari et al Nature 454 1096 2008; idea: Mateo 1992

These are essentially $v^2 r$ masses



Walker et al, ApJ in press,
0906.0341

Mamon & Boue 0906.4971

Wolf et al 0908.2995

$M < r$

- Illingworth
1976
-
- Mateo 1990s
- Strigari,
Walker,
Mamon,
Wolf...
- Many more

 $M = L?$

- Mateo et al
1990s
- Wilkinson et al
2002
- Koch et al
- Lokas
- Many more

 $M(r)$

- MB, BE, FD, RJ....
 - Eddington, Jeans,
Fricke, Chandrasekhar,
Miyamoto, Nagai,
Toomre, Lynden-Bell,
Dehnen, deZeeuw,
Evans, Kent & Gunn,
Merrifield & Kent,
Kuijken & Gilmore,
Wilkinson & KEG,
Wu & Tremaine,
Lokas.....
- Hundreds of others

From kinematics to dynamics: Jeans equation, mass modelling

- Relates spatial distribution of stars and their velocity dispersion tensor to underlying mass profile

$$M(r) = -\frac{r^2}{G} \left(\frac{1}{\nu} \frac{d\nu\sigma_r^2}{dr} + 2 \frac{\beta\sigma_r^2}{r} \right)$$

- ◆ Either (ii) determine mass profile from projected dispersion profile, with assumed isotropy, and smooth functional fit to the light profile
- ◆ Or (iii) assume a parameterised mass model $M(r)$ and velocity dispersion anisotropy $\beta(r)$ and fit dispersion profile to find best forms of these (for fixed light profile)

Core properties: adding anisotropy

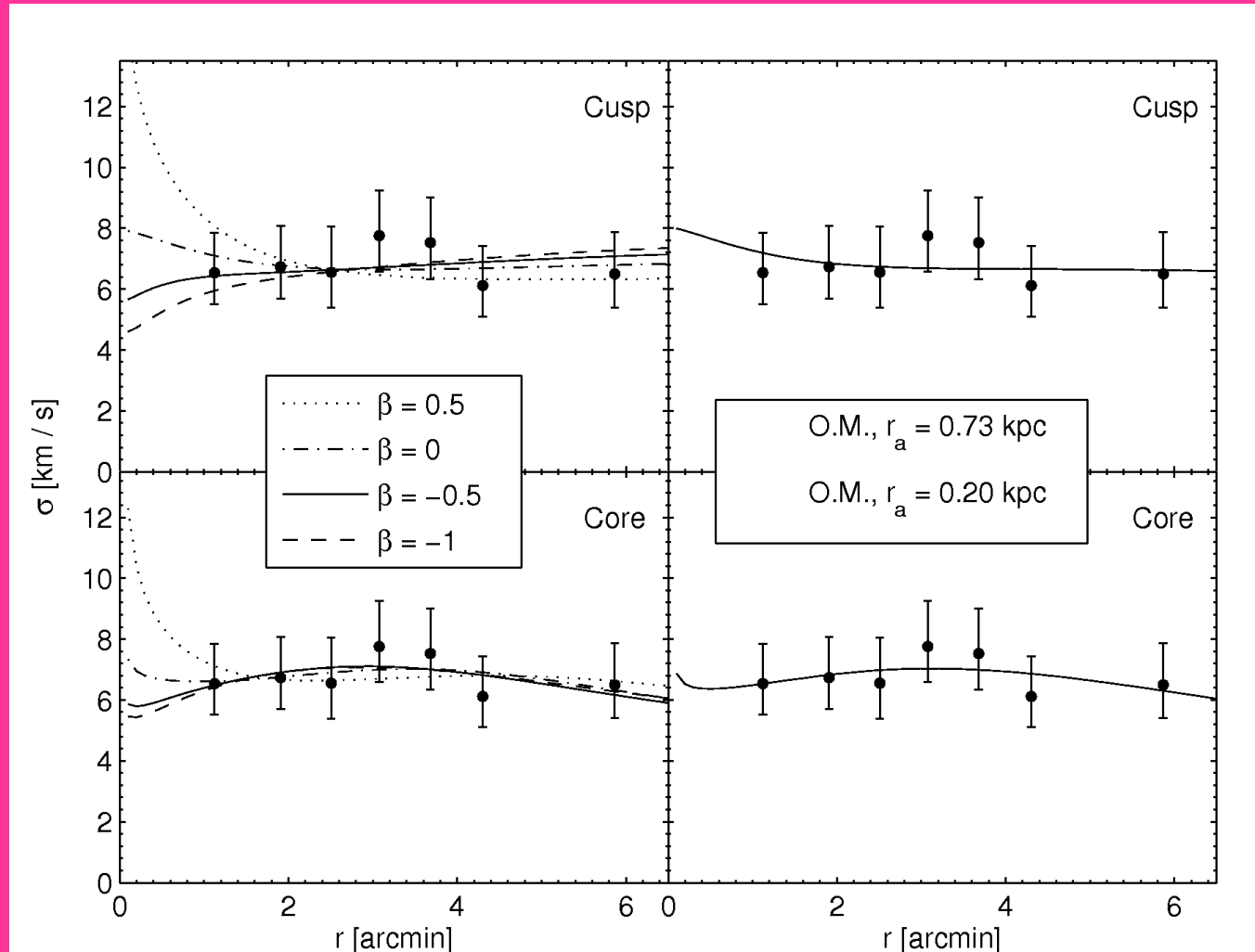
Koch et al 07
AJ 134 566 '07

Fixed β

Radially varying β

Leo II

$$\beta = 1 - \frac{\langle v_{\theta}^2 \rangle}{\langle v_r^2 \rangle}$$

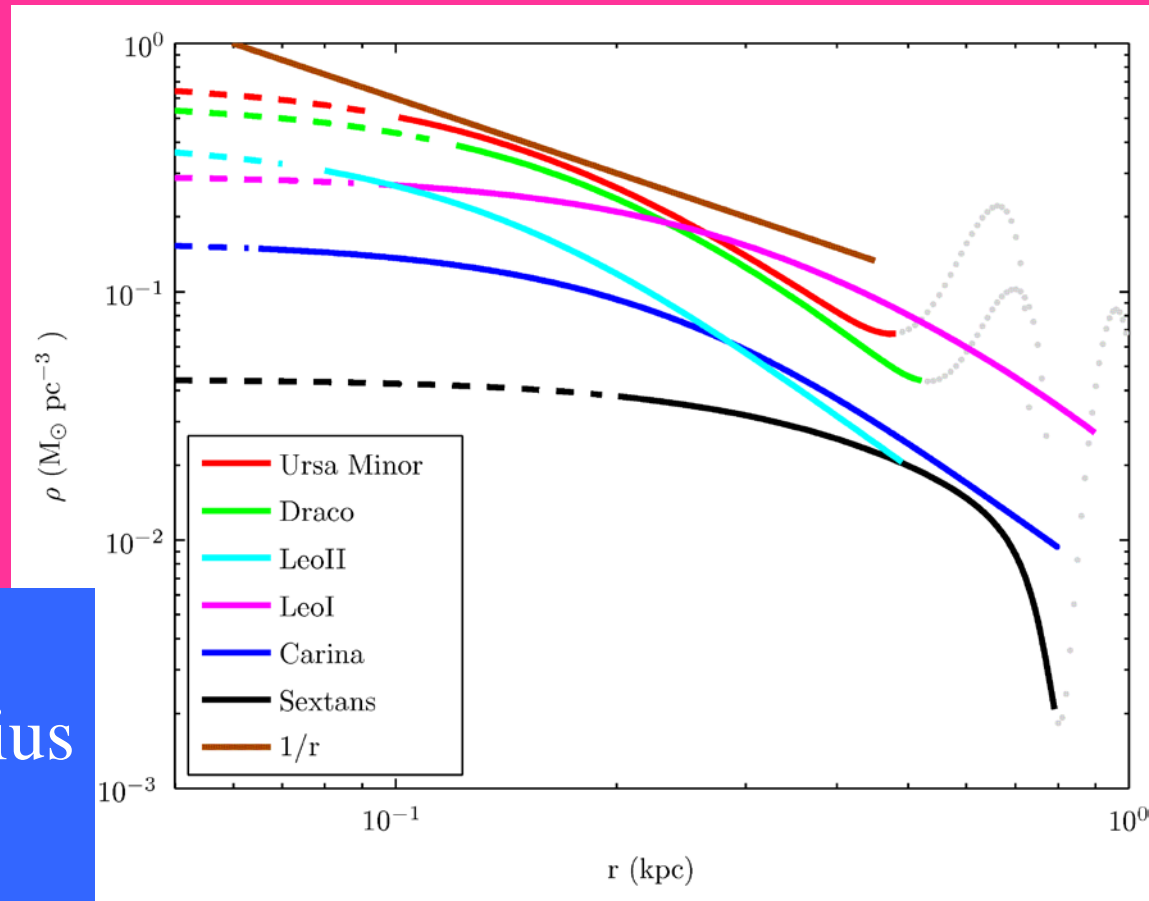


Core slightly favoured, but not conclusive: cf Lokas 2002/5

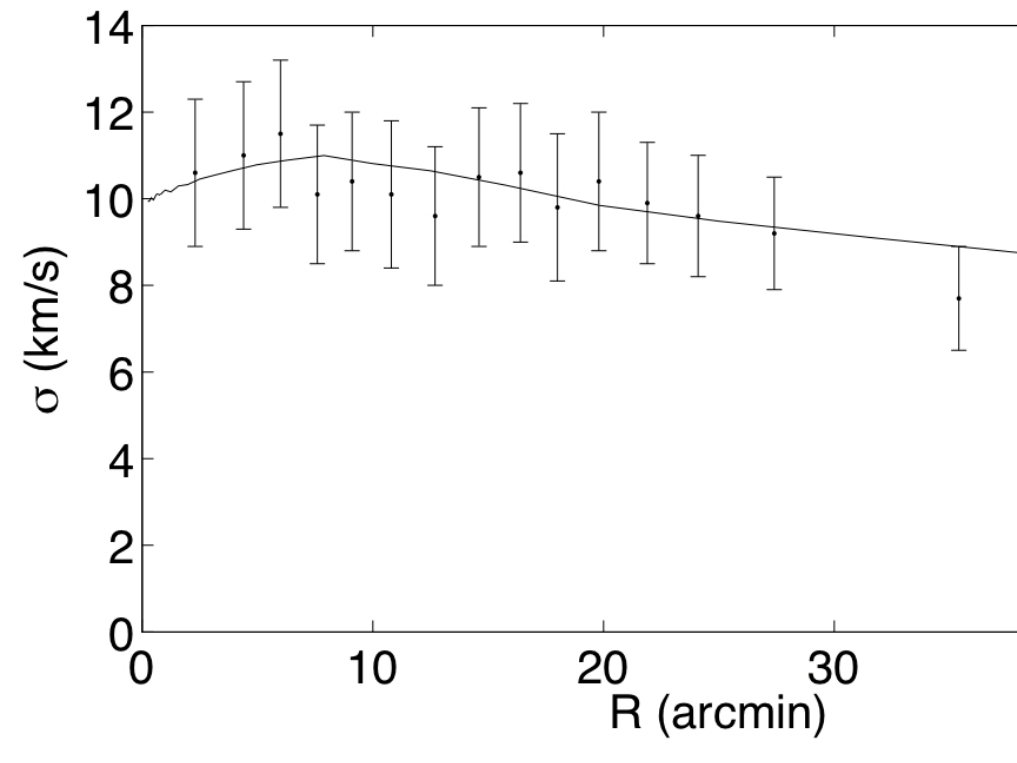
Derived mass density profiles:

Jeans' equation with assumed isotropic velocity dispersion: all consistent with cores.

CDM predicts slope of -1.3 at 1% of virial radius and asymptotes to -1 (Diemand et al. 04)



NB these Jeans' models are to provide the most objective sample comparison – DF fitted models agree with these

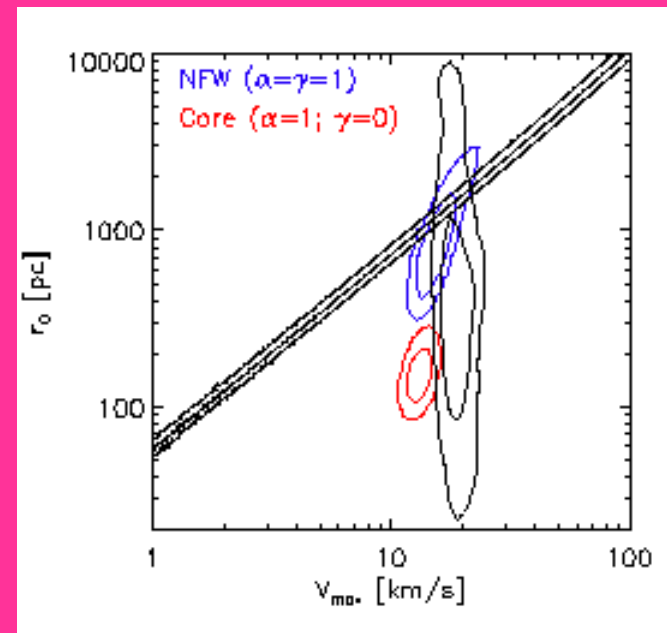
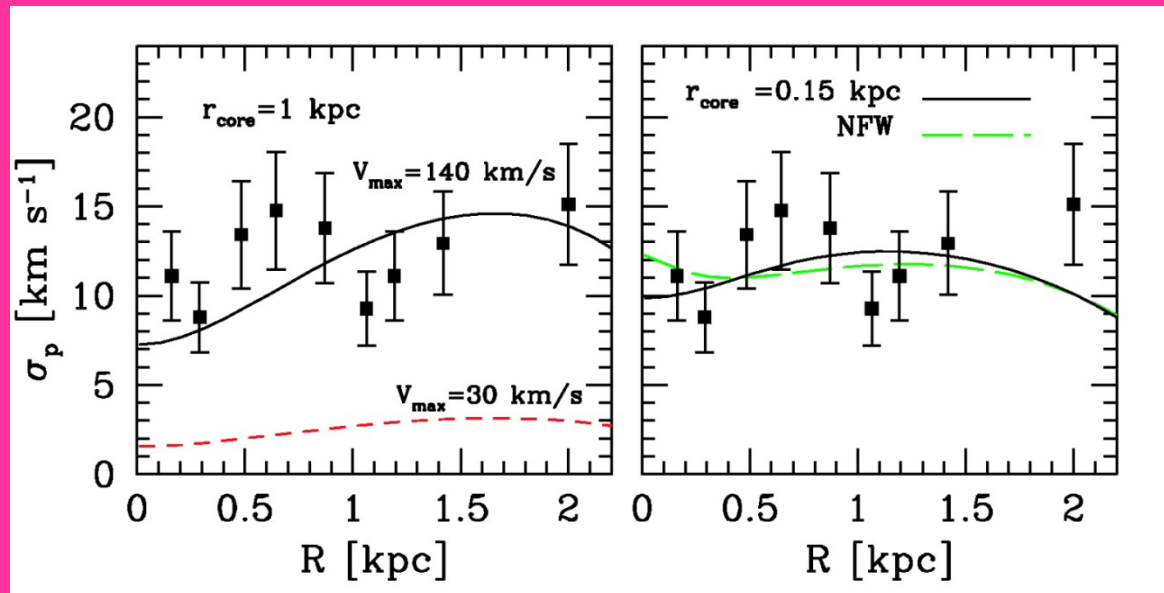


Note the data quality improvement:
 First dSph declining dispersion profile

→ Robust $V_{\max} = 20 \pm 4$ km/s

Top Walker et al 2009

Lower Strigari et al 2006 fit to
 Walker et al 2006



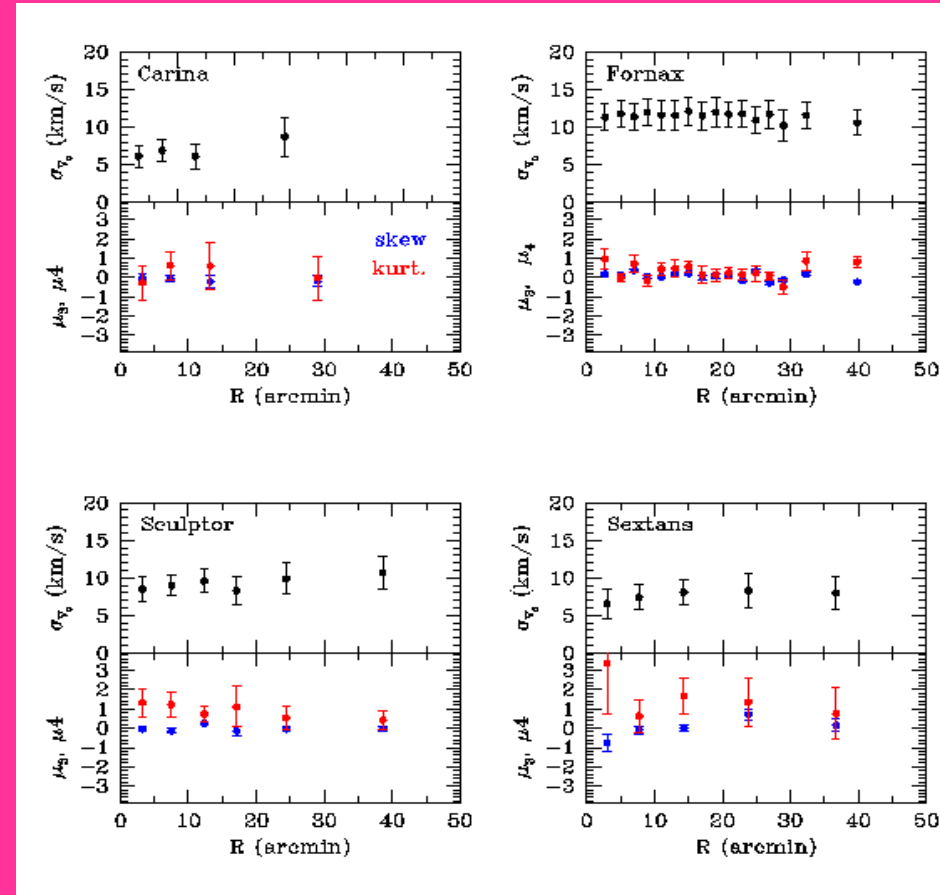
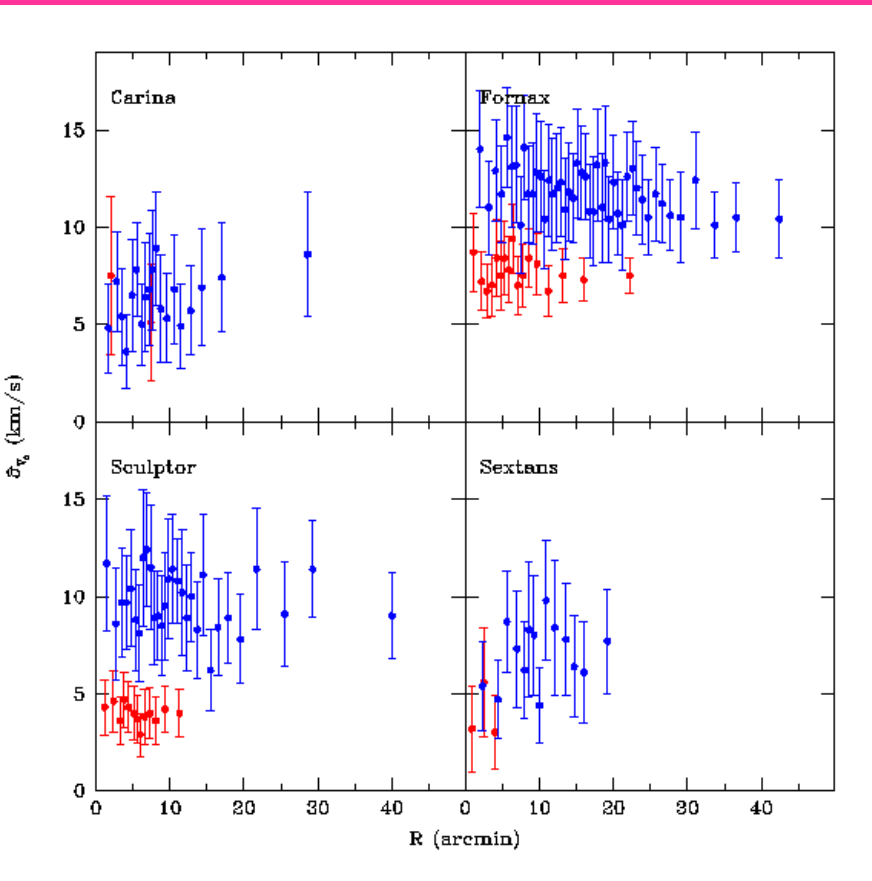
FOURTH MOMENTS AND THE DYNAMICS OF SPHERICAL SYSTEMS

MICHAEL R. MERRIFIELD AND STEPHEN M. KENT^{a)}

Analysis of the projected density and line-of-sight velocity dispersion of spherical systems leaves significant ambiguity as to the underlying dynamics. Using the additional information in the fourth moment of the line-of-sight velocity distribution is shown to impose tighter limits on the range of dynamical models that are consistent with observation. If the gravitational potential is known, the fourth moment can be deprojected uniquely to yield the three intrinsic components via formulas analogous to the second moment deprojection. Two fourth moment analogs of the virial theorem follow from this result and provide additional constraints on the form of the gravitational potential. A simple application of these formulas for testing the validity of assuming a constant mass-to-light ratio is presented. Extension of these fourth moment results to successively higher orders would imply that the form of the underlying potential and the distribution of orbits are uniquely specified by quantities that are, in principle, observable. Investigation of the fourth moment of velocity data from the globular cluster ω Cen demonstrates that useful constraints can be obtained from a dataset of only ~ 300 velocities. The fourth moment of this sample is shown to be inconsistent with an isotropic velocity distribution and suggests that the orbits become predominantly radial at large radii. A more complete analysis of this and other systems awaits larger and more homogeneous samples.

This methodology developed extensively by Lokas 2002 and fit to mass follows light models by ``outlier clipping``.

Limitation of smooth anisotropy models – DF may be complex, β multi-modal – eg central young pops?



$M < r$

- Illingworth
1976
-
- Mateo 1990s
- Strigari,
Walker,
Mamon,
Wolf...
- Many more

 $M = L?$

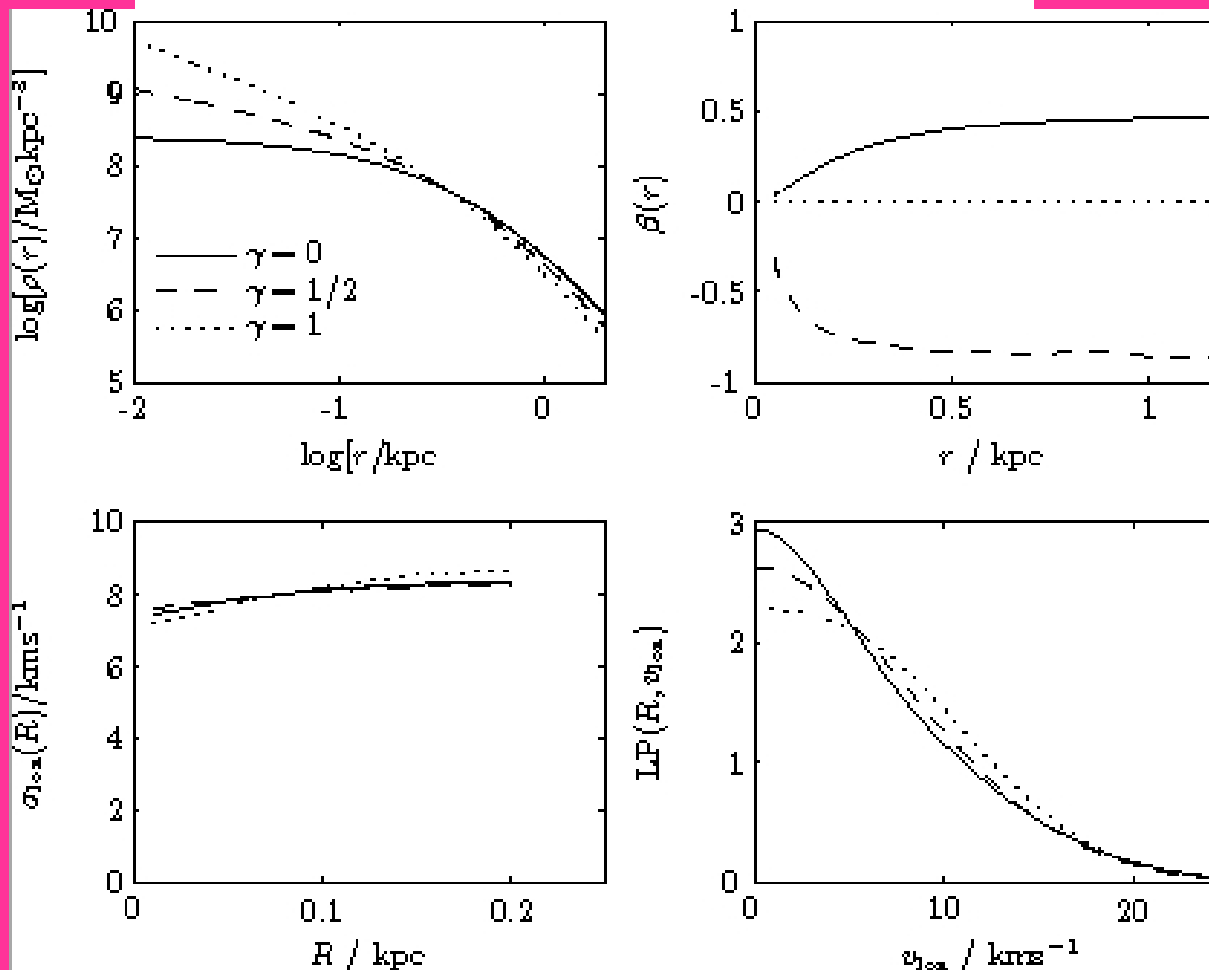
- Mateo et al
1990s
- Wilkinson et al
2002
- Koch et al
- Lokas
- Many more

 $M(r)$

- MB, BE, FD, RJ....
 - Eddington, Jeans,
Fricke, Chandrasekhar,
Miyamoto, Nagai,
Toomre, Lynden-Bell,
Dehnen, deZeeuw,
Evans, Kent & Gunn,
Merrifield & Kent,
Kuijken & Gilmore,
Wilkinson & KEG,
Wu & Tremaine,
Lokas.....
- Hundreds of others

From kinematics to dynamics: anisotropy vs mass profile degeneracy

$$M(r) = -\frac{r^2}{G} \left(\frac{1}{\nu} \frac{d\nu\sigma_r^2}{dr} + 2 \frac{\beta\sigma_r^2}{r} \right)$$



Mass measurements: DF models

This is classical physics – eg Kent & Gunn 1982

$$f_1 = A_1(e^{-E/\sigma^2} - 1)e^{-J^2/2J_0^2}, \quad (2a)$$

$$f_2 = A_2(-E)^\beta e^{-J^2/2J_0^2}, \quad (2b)$$

$$f_2 = A_3(e^{-E/\sigma^2} - 1)J^{-\gamma}, \quad (2c)$$

$$f_4 = A_4(-E)^\beta J^{-\gamma}. \quad (2d)$$

It is seen that these functions simply combine two possible forms each for the energy and angular-momentum dependence. For the energy dependence we allow for either a lowered Gaussian (with characteristic energy σ^2) or a polytrope (with power-law index β). For the angular-momentum dependence we allow for two extreme cases of anisotropy. The term $\exp(-J^2/2J_0^2)$ produces models with orbits that are isotropic in the center and radial at the edge; J_0 is the cutoff angular momentum. The term $J^{-\gamma}$ produces a more uniform anisotropy, and in fact yields a constant ratio of tangential to radial velocity dispersions (which depends on the parameter γ). Function f_1 is a King-Michie distribution, first introduced by Michie (1963) to describe the structure of globular clusters. In the limit $J_0 \rightarrow \infty$ the isotropic King (1966) models are recovered. The isotropic forms of either f_2 or f_4 ($J_0 \rightarrow \infty, \gamma \rightarrow 0$) yield standard polytropes of index $n = \beta + 3/2$ (Chandrasekhar 1939).

cf Kuijken & Gilmore (1989) for application to local DM density

cf Wu & Tremaine 2006
Wu 2007

Lokas 2002, 2005

Wilkinson et al 2002

for dSph applications

The mass and anisotropy profiles of galaxy clusters from the projected phase-space density: testing the method on simulated data

MN in press

Radosław Wojtak,^{1*} Ewa L. Łokas,¹ Gary A. Mamon^{2,3} and Stefan Gottlöber⁴

ABSTRACT

We present a new method of constraining the mass and velocity anisotropy profiles of galaxy clusters from kinematic data. The method is based on a model of the phase-space density, which allows the anisotropy to vary with radius between two asymptotic values. The characteristic scale of transition between these asymptotes is fixed and tuned to a typical anisotropy profile resulting from cosmological simulations. The model is parametrized by two values of anisotropy, at the centre of the cluster and at infinity, and two parameters of the NFW density profile, the scale radius and the scale mass. In order to test the performance of the method in reconstructing the true cluster parameters, we analyse mock kinematic data for 20 relaxed galaxy clusters generated from a cosmological simulation of the standard Λ cold dark matter model. We use Bayesian methods of inference and the analysis is carried out following the Markov Chain Monte Carlo approach. The parameters of the mass profile are reproduced quite well, but we note that the mass is typically underestimated by 15 per cent, probably due to the presence of small velocity substructures. The constraints on the anisotropy profile for a single cluster are in general barely conclusive. Although the central asymptotic value is

Very elegant, 3D – assume potential, fit anisotropy, vice versa

Next step – deduce DF and potential from data

Models

$$\rho_{\text{halo}}(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^\gamma \left(1 + \left(\frac{r}{r_s}\right)^{1/\alpha}\right)^{\alpha(\beta-\gamma)}}$$

$$\Sigma_*(R) = 2 \int_R^\infty \frac{\rho_*(r)r dr}{\sqrt{r^2 - R^2}}$$

Same form used for both halo and stars, but stellar parameters held fixed

Zhao model = generalised Hernquist/NFW/...

2-integral DF parameterised following Gerhard 1991

Constructing the line of sight velocity distributions

- Fit surface brightness profile
- Use method by P. Saha to invert integral equation for DF:

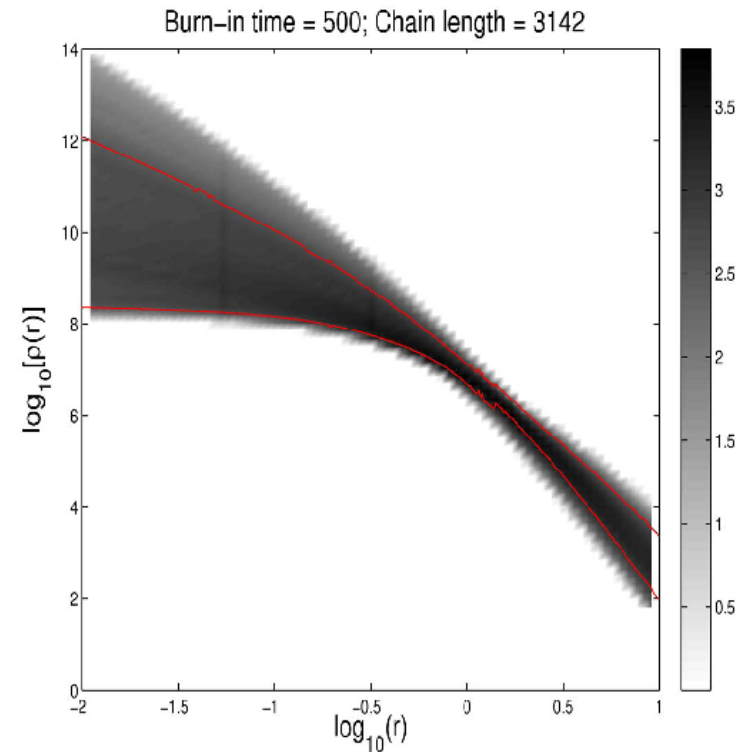
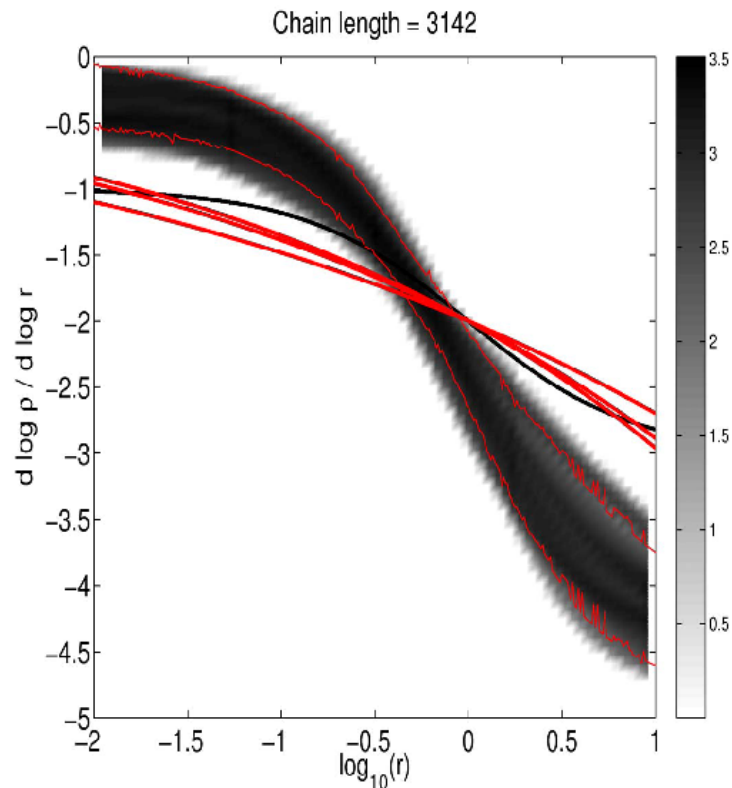
$$\rho(\Phi) = \frac{4\pi}{r^2} \int_0^\Phi w(E) dE \int_0^{L_{\max}} \frac{g(E, L)L dL}{\sqrt{2(\Phi - E) - L^2/r^2}}$$
$$L_{\max} = \sqrt{2(\Phi - E)r}$$

- Project to obtain LOS velocity distribution on a grid of R and v_{los}
- Spline to required radii for observed stars, and convolve with individual velocity errors

Fitting the kinematic data

- Surface brightness profile determined from metal-poor data (v. similar to overall profile of Fornax)
- Markov-Chain-Monte-Carlo used to scan parameter space
- Parameters: 3 velocity distribution parameters
(a, c, L_0); 4 halo parameters ($\alpha, \beta, \gamma, \rho_0$)
- Multiple starting points for MCMC used - chains run in parallel and combined once “converged”
- Error convolution included - using only data with
 $\Delta v_{\text{los}} < 2 \text{ km s}^{-1}$

Preliminary Fornax MCMC DF modelling

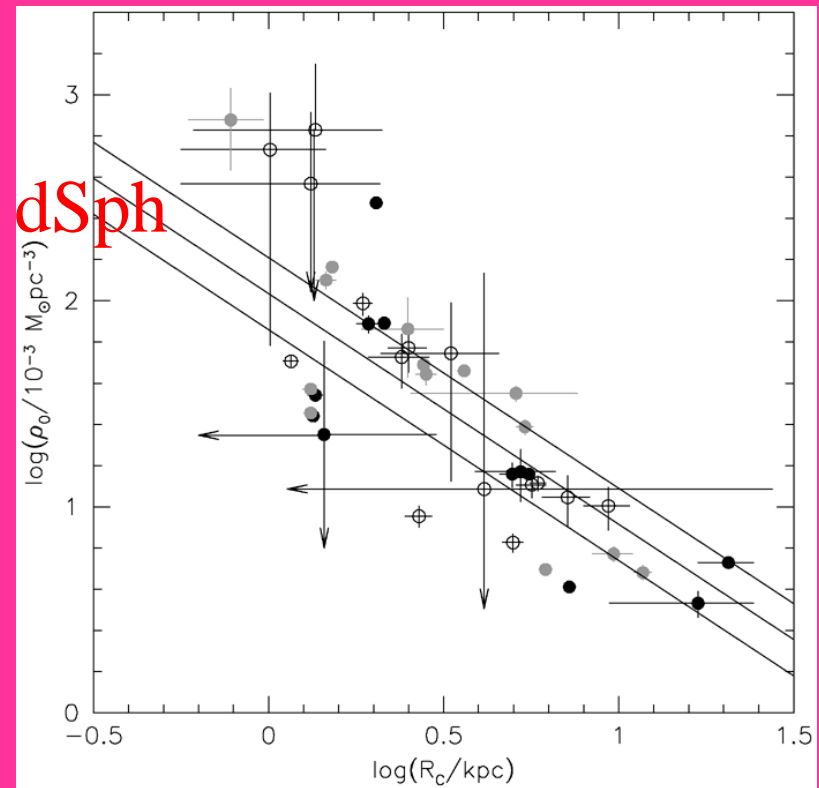
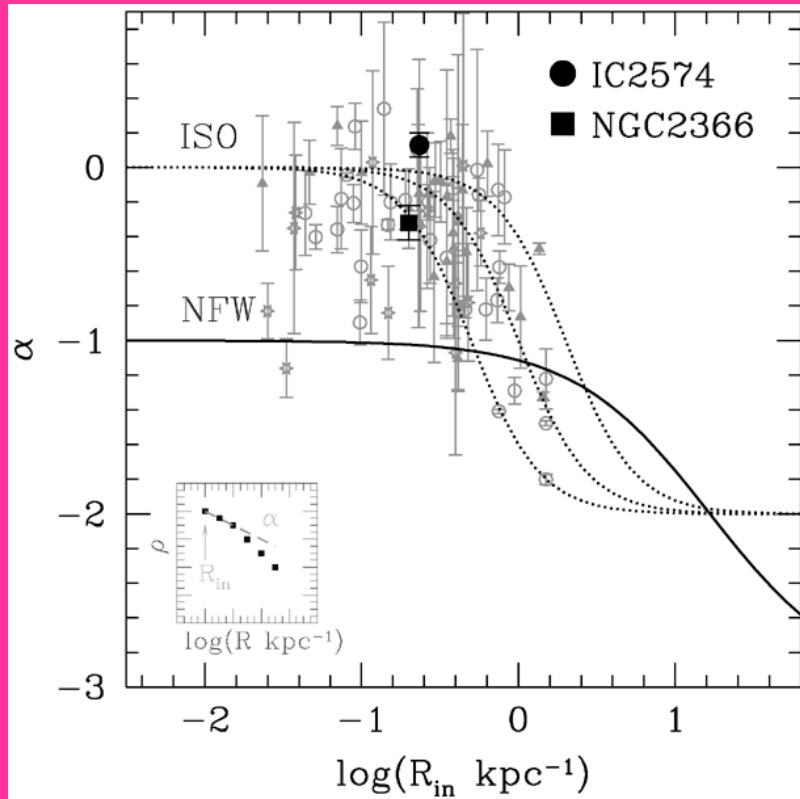


Still lots of tests and checks underway, but it shows full DF models can deduce real 3-D mass profiles
Results very similar to LSB disk (THINGS) results

Conclusion

- High-quality kinematic data exist
 - Jeans analyses → mildly prefer cored mass profiles
 - Mass-anisotropy degeneracy allows cusps in β models
 - Substructure, dynamical friction → prefers cores
 - Moment and parameter models OK for small datasets
if restrict to radii where data exist!
 - More sophisticated DF analyses are feasible, and look promising for real analyses
-
- **Cores always preferred, but not always required**
 - **Central densities always similar and low**
 - **promising results from available DF analyses**
-
- extending analysis to lower luminosity systems is difficult: small number of stars, serious tidal effects
 - One galaxy – Fornax – has a real $V_m=20\text{km/s}$

'Things' HI/Spitzer/Galex survey -- low-mass spirals consistent



Oh et al; de Blok et al AJ 2008 v136 2761; 2648

Distribution function

$$F(E, L) = w(E)g(E, L) \quad \text{Gerhard (1991)}$$

$$g(E, L) = \begin{cases} c + (1 - c)(1 - (1 - x^2))^a & \text{tangential} \\ c + (1 - c)(1 - x^2)^a & \text{radial} \end{cases}$$

$$x(E, L) = \frac{L}{L_0 + L_{\text{circ}}(E)}$$