

# Local plane coordinates for the detailed analysis of complex Gaia sources

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ABSTRACT. The detailed analysis of a complex object, such as a partially resolved visual binary, may be simplified if the observations are referred to a local coordinate system after taking into account the satellite attitude, instrument geometry, aberration and gravitational deflection. Thus these effects need not be further considered in the analysis. This is possible, to microarcsec accuracy, within a radius of  $\simeq 1$  arcmin from a chosen reference point. The definition of such local plane coordinates is given, together with a specification of the data that need to be provided with the individual observations. The advantage of the method is that the object can be analyzed without access to attitude, geometric calibration and orbit data, and without need for the corresponding transformations. A disadvantage is that more data may be needed per object. This second issue of GAIA-LL-061 makes the concept more complete compared to issue 1 of 9 October 2005.

## 1 Introduction

The detailed analysis of window (sample) or elementary (centroid) data for a non-single object is complicated by the fact that the individual observations only make sense in relation to certain attitude, calibration, and orbital data. These auxiliary data may be given in several separate, possibly quite large data sets, and their proper use requires a long sequence of transformations that are not really central to the specific object analysis problem at hand.

At the level of the object processing (CU4) and 2-d imaging (CU5), these auxiliary data are known from the astrometric GIS and other core processes, and it would seem like a good idea to ‘correct’ the window/elementary observations for these (known) effects, before handing them over to the object processing. This note describes a possible procedure for this. If and how it should actually be used is briefly discussed in Sect. 6.

The basic idea is to define for each object a local plane coordinate system in the tangent plane of the unit sphere, in which the *coordinate direction* of the object is modelled. The local system is uniquely defined by the chosen celestial coordinates of the tangent point  $(\alpha_0, \delta_0)$ . The local coordinates are similar to the ‘standard coordinates’ customarily used in small-angle astrometry (e.g., [2], [4]), basically obtained through a gnomonic (central) projection onto the tangent plane. For the individual observations, data are referred to axes oriented according to the local scan direction.

An important constraint for such local coordinates to be practically useful is that the differences between coordinate directions and observed (proper) directions are locally negligible after allowing for a change of origin and scale. This limits the size of the area in which local coordinates can be used, as discussed in the next section.

## 2 Differential effects of aberration and light deflection

To first order, stellar aberration causes the apparent position of an object to be displaced by the small angle  $(v/c)\sin\psi$  towards the direction of motion (apex), where  $v$  is the speed of the observer,  $c$  the speed of light, and  $\psi$  the angle of the object from the apex point. It is easily seen that, within a small area around the object, the differential effect amounts to an isotropic change of angular scale by the factor  $1 - (v/c)\cos\psi$ . Thus, an object located near the apex appears smaller by the (linear) factor  $1 - (v/c)$ , while an object near antapex appears magnified by the factor  $1 + (v/c)$ . Since the differential effect is isotropic, there is no distortion of the image: a circular object remains circular.

The barycentric velocity of Gaia is always around  $30\text{ km s}^{-1}$ , or  $v/c \simeq 10^{-4}$ . The apparent magnification factor consequently varies between 0.9999 and 1.0001. At Gaia accuracies, this is significant even for ‘small’ objects; e.g., in a binary with component separation 1 arcsec, the apparent separation may vary by  $\pm 100\ \mu\text{as}$  (it also affects the apparent flux by some fraction of a millimag). Thus, e.g., if the object *appears* smaller, the *observed* proper direction offsets have to be *increased* in the transformation to coordinate direction offsets.

On a much larger angular scale, the differential effect obviously cannot be isotropic, so the representation by a single magnification factor breaks down at a certain radius. From numerical tests using the full relativistic aberration formula (Lorentz transformation), it is found that, for  $v = 30\text{ km s}^{-1}$ , the simple magnification model is accurate to  $\leq 1\ \mu\text{as}$  within a radius of  $\simeq 1$  arcmin. This holds even for the most unfavourable case of  $\psi \simeq 90^\circ$ . The residual error increases quadratically with the radius.

There are other effects besides aberration that must be considered when defining the local plane coordinates, the most important being gravitational deflection by the Sun. This causes an apparent shift by approximately  $(4\text{ mas})\cot(\psi/2)$  away from the Sun, where  $\psi$  is now the angle of the object from the Sun. In contrast to the aberration, the effect is anisotropic: the image is always compressed along the circle through the object and the Sun, and somewhat less compressed (or, for  $\psi < 90^\circ$ , magnified) in the perpendicular direction. Thus, differential light deflection cannot be described by a single scale factor. Fortunately, the effect is much smaller than for the aberration, and does not exceed  $1\ \mu\text{as}$  within a radius of 19 arcsec in the most unfavourable case of  $\psi = 45^\circ$  (the smallest Sun angle allowed by the scanning law). The residual effect increases linearly with radius, and may in practice be neglected ( $< 3\ \mu\text{as}$ ) within a radius of 1 arcmin.

To sufficient accuracy for the Gaia object processing, the differential effects of aberration and light deflection by the Sun can therefore always be represented by a simple change of scale within a field of up to  $\sim 1$  arcmin radius.

There are exceptional circumstances where this model is inadequate to describe the local distortion. This may be the case near one of the major planets (Jupiter, Saturn, etc.). A possible solution could be to model the gravitational deflection near these bodies as a superposed, local effect. Moreover, the local coordinates are of course only useful for objects that stay within the 1 arcmin area for the duration of the mission; this applies to all stellar and extragalactic objects, but not to solar-system objects.

The use of local plane coordinates, in which differential aberration is accounted for by a

change of scale, was originally adopted for the NDAC processing of double stars [3].

### 3 Transformations involving local coordinates

For the reference point  $(\alpha_0, \delta_0)$  we define the reference triad  $[\mathbf{p}_0 \ \mathbf{q}_0 \ \mathbf{r}_0]$  by means of the three orthogonal unit vectors

$$\mathbf{p}_0 = \begin{bmatrix} -\sin \alpha_0 \\ \cos \alpha_0 \\ 0 \end{bmatrix}, \quad \mathbf{q}_0 = \begin{bmatrix} -\sin \delta_0 \cos \alpha_0 \\ -\sin \delta_0 \sin \alpha_0 \\ \cos \delta_0 \end{bmatrix}, \quad \mathbf{r}_0 = \begin{bmatrix} \cos \delta_0 \cos \alpha_0 \\ \cos \delta_0 \sin \alpha_0 \\ \sin \delta_0 \end{bmatrix}. \quad (1)$$

$\mathbf{r}_0$  is the direction to the reference point, while  $\mathbf{p}_0$  indicates local ‘East’ (increasing  $\alpha$ ) and  $\mathbf{q}_0$  local ‘North’ (increasing  $\delta$ ) if  $|\delta_0| < 90^\circ$ . Note however that the reference triad is perfectly well-defined also at the celestial poles, where  $\alpha_0$  remains significant.<sup>1</sup>

For an arbitrary coordinate direction  $\mathbf{c}$  (in the vicinity of  $\mathbf{r}_0$ , so that  $\mathbf{r}'_0 \mathbf{c} > 0$ ) local plane coordinates  $(a, d)$  are usually defined through gnomonic projection<sup>2</sup>

$$a = \frac{\mathbf{p}'_0 \mathbf{c}}{\mathbf{r}'_0 \mathbf{c}}, \quad d = \frac{\mathbf{q}'_0 \mathbf{c}}{\mathbf{r}'_0 \mathbf{c}}. \quad (2)$$

The inverse relation is

$$\mathbf{c} = \frac{\mathbf{r}_0 + \mathbf{p}_0 a + \mathbf{q}_0 d}{(1 + a^2 + d^2)^{1/2}}. \quad (3)$$

Equations (2) and (3) are formally identical to the usual transformations involving standard coordinates (e.g., [4]), except for the purely practically motivated absence of the denominators. However, we make the important distinction that  $\mathbf{c}$  must always be interpreted as a *coordinate direction*, not as an observed (proper) direction. This guarantees that the object modelling can be completely carried out in  $(a, d)$ . For example, the combination of proper motion, parallax and a Keplerian orbit (using Thiele–Innes elements) can be parameterized as

$$\left. \begin{aligned} a(t) &= a_T + (t - T)\mu_{\alpha^*} + f_a(t)\pi + BX(t) + GY(t) \\ d(t) &= d_T + (t - T)\mu_\delta + f_d(t)\pi + AX(t) + FY(t) \end{aligned} \right\} \quad (4)$$

where  $(a_T, d_T)$  is the offset at epoch  $T$  and  $f_a, f_d$  are the known parallax factors, etc.

In order to express observations as simply as possible in the local system, we make however one further transformation, viz., from the local *celestial* coordinates  $(a, d)$  to the local *scan* coordinates  $(w, z)$  (Fig. 1). This transformation is completely determined by  $\theta$ , the position angle of the scan:

$$\left. \begin{aligned} w &= a \sin \theta + d \cos \theta \\ z &= -a \cos \theta + d \sin \theta \end{aligned} \right\}. \quad (5)$$

<sup>1</sup>For example, the two reference points  $(\alpha_0, \delta_0) = (0^\circ, 90^\circ)$  and  $(\alpha_0, \delta_0) = (90^\circ, 90^\circ)$  are not equivalent: they define different reference triads.

<sup>2</sup>The usual notation for standard coordinates is  $(\xi, \eta)$ , but since  $\eta$  is used for the along-scan field angle,  $(a, d)$  is here suggested for the local plane coordinates.  $a$  and  $d$  roughly correspond to  $\Delta\alpha \cos \delta$ ,  $\Delta\delta$ .

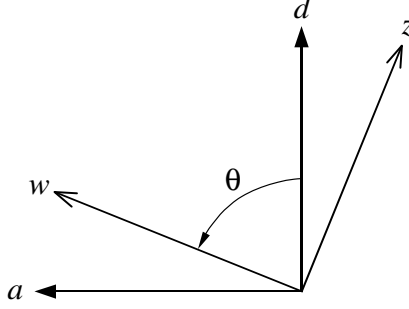


FIGURE 1: The transformation between local celestial coordinates  $(a, d)$  and local scan coordinates  $(w, z)$  is uniquely given by the position angle of the scan,  $\theta$ . The figure indicates the sense of directions as seen from the centre of the celestial sphere.

Conversely,

$$\left. \begin{aligned} a &= w \sin \theta - z \cos \theta \\ d &= w \cos \theta + z \sin \theta \end{aligned} \right\}. \quad (6)$$

Loosely speaking,  $+w$  is the local direction in which the FOV moves on the sky (the AL direction in the local system), while  $+z$  is the local AC direction (with the Sun at  $z > 0$ ). The precise definition, however, is the following:  $+w$  is the local direction of increasing field coordinate  $\eta$ ;  $+z$  is the local direction of increasing field coordinate  $\zeta$ . The procedure for calculating  $\theta$  for a particular observation shall be detailed elsewhere. It involves nothing but the attitude and the instantaneous field coordinates  $(\eta, \zeta)$  of the celestial object under consideration.

The use of  $(w, z)$  instead of  $(a, d)$  allows to distinguish easily between AL and AC quantities. For example, in many cases only the AL coordinate is of interest, and we may then omit  $z$  altogether. Also, the observational errors are normally uncorrelated between  $w$  and  $z$  (and usually much larger in  $z$  than in  $w$ ), whereas a strong correlation would usually be found between  $a$  and  $d$ . The specification of samples, patches and windows is also naturally related to  $(w, z)$ .

The transformation of an arbitrary object model to the local scan coordinates is straightforward; for example, for the model in (4) we obtain

$$\left. \begin{aligned} w &= sa_T + cd_T + (t - T)s\mu_{\alpha^*} + (t - T)c\mu_\delta + f_w\pi + XcA + XsB + YcF + YsG \\ z &= -ca_T + sd_T - (t - T)c\mu_{\alpha^*} + (t - T)s\mu_\delta + f_z\pi + XsA - XcB + YsF - YcG \end{aligned} \right\} \quad (7)$$

where, for brevity, we have put  $s = \sin \theta$ ,  $c = \cos \theta$  and introduced the parallax factors in local scan coordinates,

$$\left. \begin{aligned} f_w &= f_a \sin \theta + f_d \cos \theta \\ f_z &= -f_a \cos \theta + f_d \sin \theta \end{aligned} \right\}. \quad (8)$$

## 4 Expressing observations in local coordinates

### 4.1 Elementary observations (centroid positions)

The outcome of the image centroiding process is a determination of the accurate time  $t$  when the image centre was transferred to the read-out register, interpolated to sub-pixel resolution. This is the main along-scan (AL) astrometric observation. There is also an across-scan (AC) coordinate, given by the CCD number ( $n$ ) and pixel column number ( $m$ ) [1], which for the SM will also always be interpolated to sub-pixel resolution. For the AF it can be measured only in exceptional cases (bright stars and special calibration modes) due to the on-chip binning in the AC direction. The geometrical calibration of the CCDs provides the mapping from  $(n, m)$  and FOV index to field angles  $(\eta, \zeta)$ , which combined with the attitude gives the observed (proper) direction  $\mathbf{u}$  to the object at the instant  $t$ . Removing aberration and gravitational light deflection gives the coordinate direction  $\mathbf{c}$  at time  $t$ , from which  $(a, d)$  are computed by means of (2) and finally  $(w, z)$  by (5).<sup>3</sup>

Note that for these transformations the knowledge of the across-scan field coordinate  $\zeta$  is needed. Since this cannot be derived directly for the vast majority of the AF measurements. In these cases it must thus either be extrapolated from the corresponding measured SM centroid or from prior knowledge of the object’s astrometric parameters. Appendix A shows that integer-pixel precision is not sufficient for the purpose.

Now let  $\sigma_w = \sigma_\eta$  and  $\sigma_z = \sigma_\zeta$  be the standard errors of the centroiding AL and AC, respectively, expressed as angles; then the complete specification of the elementary astrometric observation is:  $(t, \theta, w, \sigma_w, z, \sigma_z, f_w, f_z)$ , where  $z, \sigma_z$  can be omitted for a purely one-dimensional (AL) observation (at least  $\sigma_z$  will not make much sense). Note, however, that we added the parallax factors  $(f_w, f_z)$ , so that one does not need a satellite ephemeris to interpret the observations. In addition, the reference position  $(\alpha_0, \delta_0)$  must of course be specified; this would be the same for all observations of a given object.

We herewith define the reference position  $(\alpha_0, \delta_0)$  to be the catalogue position (i.e. the position at the catalogue epoch) in the astrometric source catalogue file belonging to the same delivery of the Gaia Main Database as the local plane coordinates file under consideration. This definition is always unambiguous, and it is in accordance with the ‘versioning’ concept for the overall data flow in the Gaia data reduction. It also does not entail any extra data flow or data organization, since the astrometric parameters of any object will always be needed for object processing.

In the original data  $(t, n, m)$ , the AL positional information was essentially provided by the time  $t$ , which therefore had to be given with a resolution of some nanoseconds.<sup>4</sup> When transformed to local scan coordinates, the AL positional information is instead given by  $w$ , and  $t$  is only needed to the moderate precision determined by the object’s motion or variability. For example, a common value for  $t$  may suffice for each FOV transit (not for planetary objects!), and the sequence of  $(w, z)$  values for the individual CCD transits could similarly be condensed, by averaging, to a single observation for each FOV transit.<sup>5</sup>

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<sup>3</sup>This computation can be simplified by skipping the intermediate coordinates  $(a, d)$

<sup>4</sup>Double precision is not sufficient to express time to that resolution over a sufficiently long period. A suitable format for absolute time scales could instead be the number of nanoseconds from J2000.0, expressed as a signed long integer (64 bits). The wrap-around time is  $2^{63}$  ns = 292 years.

<sup>5</sup>During the discussions of issue 1 of the present document, both D. Pourbaix (for CU4) and U. Bastian

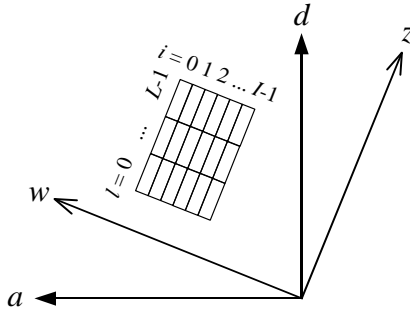


FIGURE 2: Specification of a window of  $I \times L$  samples in local scan coordinates  $(w, z)$ .

Table 1 shows a possible specification of the input data per object.

Note that photometric data are not included in the table, although it would be natural to include at least the G magnitudes (calibrated, and probably averaged over all AF strips) and some RP/BP colour information for each FOV transit in the same structure. This inclusion was indeed requested by CU4 during the discussion of version 1 of the present document. Of course, CU4 is well justified to request a smooth and easy-to-use combination of astrometric and photometric data. However, the present document intends to specify the conceptual interface between Gaia’s core astrometry and its users. The combination to other data should be an issue in the compilation of the Interface Control Document for the Gaia Main Database, into which the whole concept of the present document has to enter eventually, too.

## 4.2 Window data (samples), simplified model

The local scan coordinates are convenient also for specifying in an absolute sense the celestial location of any sample, patch or window in the CCD data stream. Consider the fairly general case of a window containing  $I \times L$  samples, as shown in Fig. 2 (one-dimensional windows are represented with  $L = 1$ ). Within the window, let the samples be indexed  $i = 0 \dots (I - 1)$  along scan and  $l = 0 \dots (L - 1)$  across scan.

For ease of reference and ease of understanding we give the original model of issue 1 in the present subsection, and a more complete one in the following subsection.

Taking the sample indexed  $(i, l) = (0, 0)$  as origin, we may compute the observed (proper) direction corresponding to its centre exactly as for the image centroid in Sect. 4.1. Removing aberration and light deflection gives the coordinate direction and hence, using (2) and (5), its local scan coordinates  $(w_0, z_0)$ . In principle we could repeat the calculation

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(for CU3) have argued against this averaging, because it would kill a lot of noise information and outlier treatment possibilities (cosmic rays, disturbing stars from the other field of view and so on). It also would disable CU4 to independently take the correlations between different CCD transits from the same field-of-view transit into account in their error calculus. L. Lindegren has argued to the contrary, saying that all this can be done by CU3 already, in the computation of the average and its standard error. The simplifications for the later stage (object processing) are very great, indeed. This discussion is still open.

for each sample, but it is sufficient to compute the sample dimensions  $(\Delta w, \Delta z)$  (which may be negative depending on the adopted indexing convention), from which

$$\left. \begin{aligned} w_i &= w_0 + i\Delta w & (i = 0 \dots I - 1) \\ z_l &= z_0 + l\Delta z & (l = 0 \dots L - 1) \end{aligned} \right\}. \quad (9)$$

A simple expedient (albeit perhaps not the most efficient one) could be to compute  $(w_{I-1}, z_{L-1})$  rigorously as for the first sample, and then use (9) to compute  $(\Delta w, \Delta z)$ . It should be noted that the sample dimensions vary among the observations because of differential optical distortion and differential aberration, so it will not be possible to use fixed values.

The complete specification of the window data would consist of  $t$  (the approximate time of observation);  $\theta$  (the position angle of the scan);  $I$  and  $L$  (the size of the window);  $w_0$  and  $z_0$  (the local scan coordinates of the first sample);  $\Delta w$  and  $\Delta z$  (the sample dimensions in local scan coordinates); and the sample values  $S_{il}$  for  $i = 0 \dots I - 1$  and  $l = 0 \dots L - 1$ . Again, the parallax factors  $f_w$  and  $f_z$  should be added, but they are in practice identical for all CCD transits in a given FOV transit. Table 2 shows a possible specification of the input data per object, neglecting additional information needed for photometric uses, but already including the shear terms introduced and motivated in the following subsection.

### 4.3 Window data (samples), complete model

The previous subsection mentions optical distortion and differential aberration to cause variations in the effective sample dimensions  $(\Delta w, \Delta z)$ . In fact there are more causes for such variations: Focal-length evolution, scan motion variations along scan, non-zero attitude motion across-scan. Furthermore, most of these effects do not only create changes in  $(\Delta w, \Delta z)$ , but they also lead to shear terms in (9). These shear terms are too large to be ignored, as will be shown below. A complete model for the window (sample) data would thus have to use an extension of (9) in the following form:

$$\left. \begin{aligned} w_{i,l} &= w_{0,0} + i\Delta w + lc_w & (i = 0 \dots I - 1, l = 0 \dots L - 1) \\ z_{i,l} &= z_{0,0} + l\Delta z + iv_z & (i = 0 \dots I - 1, l = 0 \dots L - 1) \end{aligned} \right\}. \quad (10)$$

In this case, the simple (albeit again perhaps not the most efficient) expedient could be to compute  $(w_0, z_0)$ ,  $(w_0, z_{L-1})$  and  $(w_{I-1}, z_0)$  rigorously, and then use (10) to compute  $(\Delta w, \Delta z, c_w, v_z)$ , see Fig. 3.

It might be instructive to briefly consider the causes of the shear terms and their probable sizes. This subject is illustrated in Fig. 4. This will be done in the rest of the present subsection. It may be skipped without disadvantages for the implementation or usage of the local plane coordinates.

The left-hand panel shows that shear terms may arise in the  $w, z$  coordinate system even if the pixels/samples form an exactly rectangular pattern on the sky. The reason is a possible rotation of the image of the CCDs on the sky with respect to the field-of-view reference system (the latter being defined by the great circle joining the two projections of some ‘central’ FPA point on the sky, in this way setting the  $(\eta, \zeta)$  field coordinates, and in consequence the  $(w, z)$  local scan coordinates). Due to the TDI operation of the CCDs,

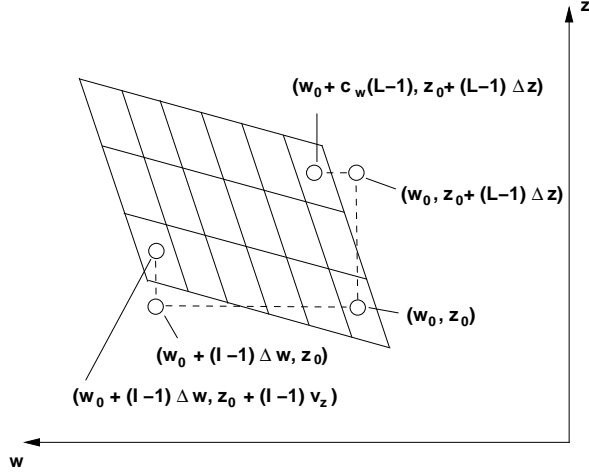


FIGURE 3: Local scan coordinates  $(w, z)$  for individual samples, with and without shear terms.

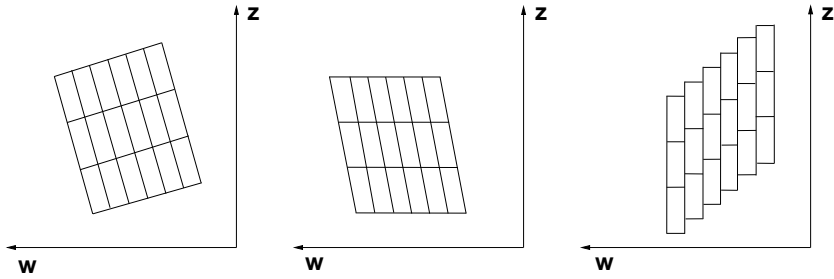


FIGURE 4: The underlying causes of the shear terms illustrated; see text.

such a rotation leads to a  $c_w$  term only, but not to a  $v_z$  term (see below). Possible causes of such a rotation are at least threefold: the imprecision of the glueing of the CCDs onto the focal-plane array (FPA), a rotation of the FPA with respect to the above-mentioned great circle (the ‘image rotation’ known from Hipparcos), and an across-scan offset of the ‘central’ FPA point from that great circle (the ‘differential image rotation’ known from Hipparcos). Appendix A shows that the effects are probably significant.

The center panel shows an actual shear of the CCD image on the sky. It may be created by optical distortion. Again, due to the TDI operation of the CCDs, this effect leads to a  $c_w$  term only, but not to a  $v_z$  term (see below).

The right-hand panel shows the effect of an across-scan attitude motion. This time the samples/pixels shown in the panel do not indicate the projection of the physical CCD samples/pixels on the sky, but the projection of TDI samples, i.e. of the actual Gaia data items. The location of a TDI sample represents the average location on the sky of the corresponding charge cloud over the actual exposure time for this sample. An across-scan attitude motion slowly shifts the location of a given CCD column in the across-scan direction on the sky, and thus the across-scan position of the TDI samples derived from that column. Across-scan attitude motion is the *only* effect that creates a  $v_z$  term.

As an aside we would like to make a remark on the ‘sample dimensions’ ( $\Delta w, \Delta z$ ). Due to the TDI operation of the CCDs, the along-scan sample dimension (i.e.  $\Delta w$ ) has no connection whatsoever with the physical size of the samples/pixels on the sky. Thus it does not change by e.g. focal-length variations, optical distortion or other optical effects. Instead, it is determined solely by the ratio between the along-scan attitude motion and the TDI time interval. It strictly measures the angle by which the spacecraft has rotated between two successive TDI clock strokes, averaged over the actual exposure time for any given star image. In stark contrast, the across-scan sample dimension (i.e.  $\Delta z$ ) is solely determined by optical effects and by the physical size of the CCD pixels, i.e. by the focal length, optical distortion, the FPA temperature and so on. It strictly measures the angular offset between neighbouring CCD columns, averaged over the path that the star image has taken on the CCD until being read out at the trailing edge of the CCD.

#### 4.4 Combining elementary and window data

Several data items appear both in Table 1 and 2, and it might be desirable to combine the two data structures (Table 3). Since most of the overlap is in the headers, the saving is moderate in terms of the total data volume.

## 5 Planetary objects

The concept of local plane coordinates as presented in this document cannot be used for solar-system objects, for at least two fundamental reasons:

1. There is no obvious choice for the reference position  $(\alpha_0, \delta_0)$ . In fact, since most of the planetary objects over the course of five years move across the whole sky, no possible choice of  $(\alpha_0, \delta_0)$  for a given object could be used throughout the whole Gaia mission.
2. For a planetary object it is impossible to compute a coordinate direction without precisely knowing the orbit. Both the (huge) parallactic effect as well as the relativistic light bending strongly depend on the distance to the object.

Therefore it seems unavoidable that the planetary-objects task has to make use of the original Gaia centroids, corresponding to proper directions. Further thinking might lead

to an interface that avoids explicit use of the Gaia calibration and attitude files, but the Gaia orbit and solar-system ephemeris are fundamentally unavoidable. It is thus doubtful whether a special astrometric interface would be worthwhile in the case of planetary objects.

As an aside we mention that in the case of planetary objects any averaging over several CCD transits should not be done. The angular motions of the objects are quite large, and also the changes of the actual scan direction over a minute of time are relevant.

## 6 Discussion

Using local coordinates as described above for the astrometric data input to the object processing, either at the elementary or window level, gives two very significant advantages:

1. It relieves the object processing and 2-d imaging workpackages from the tedious, complicated and potentially error-prone mechanics of making all the transformations and corrections involving the attitude, calibration and orbital data.
2. It provides an extremely simple and transparent data interface, where all the relevant information for a specific object may be collected in a single, small file.

Possible disadvantages are:

1. More processing is required to generate the data in local coordinates than just to copy the relevant files (elementary or raw data, plus attitude, etc). Actually, the total processing may not be much affected, since the processing is merely shifted from one place to another.
2. The data volumes to transfer may increase, since the local scan coordinates require additional information to be provided along with the data – position angle of the scan, pixel dimensions in local scan coordinates, etc. See below.
3. Definitely, similar data need to be transferred several times as the attitude and calibration data are modified in the global GIS iterations. Even the original (pixel-coordinate) centroids and the sample values will change, although less frequently.

Concerning item 2 we have to compare the data sizes in Tables 1–3 with the typical sizes of the astrometric elementary and window data per object. For the astrometric elementaries we need the time (long) and transverse pixel coordinate (int) per CCD transit, the CCD row (int) and flags (int) per FOV transit, and an identifier (long) per object; the total size (assuming  $N = 80$ ,  $M = 10$ ) is 11 kBytes per object. This is nearly 3 times the amount in Table 1, and about 1/4 of that in Table 3. For the sample data we need in addition 6 sample values (float) per CCD transit (assuming  $IL = 6$ ), increasing the size to 32 kByte per object. The saving compared to Table 3 is only 30% or 14 kByte per object. This can be further reduced if the data for the local plane coordinates are directly combined with the astrometric elementaries.

Note however that the attitude file for the whole mission (5 years) will be about 1 GByte, and the geometric calibration files will be at least of the same order. These need to be

transferred in addition to the elementary/sample data in case the local scan coordinates are not used. Thus, whether or not there is a net saving in the total data volume depends on the total number of objects that will be treated by the object processing and the 2-d imaging, and whether the transfer of the relevant data for a subset of the whole  $10^9$  Gaia objects will be practical. The break-even number is of order  $(3 \text{ GByte})/(6 \text{ kByte}) = 500\,000$ . Since it is likely that many millions of objects will be treated in the object processing task, and essentially all objects in the 2-d imaging task, the raw sample data plus attitude etc. would still give the smaller volume, but only by about 15%. The penalty would be to install a lot of additional astrometric and data handling software on the side of the object processing and 2-d imaging.

In conclusion, it is not entirely clear if and how the local scan coordinates should be used. Apart from their possible use for defining a simple data interface to the object processing, ideal e.g. for distributed GRID processing, they could also be used internally (in initial data preparation stages) in the object processing and 2-d imaging tasks. A decision should soon be taken between CU3 (GIS), CU4 (object processing) and CU5 (2-d imaging). If the decision turns out in favour of local plane coordinates, these will then have to be implemented into the Interface Control Document for the Gaia Main Database (CU1) and in the CU3 core processing environment.

## References

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## Appendix A: The need of AC field coordinates to compute AL local coordinates

The practical computation of local scan coordinates  $(w, z)$  from measured centroids  $(t, n, m)$  was briefly sketched in Section 4.1, where  $t$  is the observed transit time (also called along-scan pixel coordinate),  $n$  is the CCD number and  $m$  is the across-scan pixel coordinate. In slightly more detail this computation runs as follows:

- Step 1:  $(t, n, m) \rightarrow (t, \eta, \zeta)$ , field coordinates, using the geometric calibration
- Step 2:  $(t, \eta, \zeta) \rightarrow \mathbf{u}$ , proper direction, using the attitude
- Step 3:  $(t, \mathbf{u}) \rightarrow (t, \mathbf{c})$ , coordinate direction, using ephemerides (sun and spacecraft)
- Step 4:  $(t, \mathbf{c}) \rightarrow (t, a, d)$ , local celestial coordinates, using reference point  $(\alpha_0, \delta_0)$
- Step 5:  $(t, a, d) \rightarrow (t, w, z)$ , local scan coordinates, using attitude information.

Generally, in the AF the coordinate  $m$ , and thus  $\zeta$  is not directly known from the measurements. Therefore the above transformation steps could not be done to the necessary high precision unless these coordinates would be derived from external information. Such information can either be the measured AC coordinate of the same celestial object from the (always available) immediately preceding SM transit, extrapolated to the time of the AF transit under consideration. Alternatively it could be an approximate knowledge of the object's position from a-priori astrometric parameters (position, proper motion, parallax).

This appendix briefly looks at the question which precision the external AC information needs to have in order not to disturb the inherent precision of the AL measurements in the AF. Integer-pixel precision is not sufficient, as will be seen immediately.

Let the glueing of the CCDs onto the Gaia focal-plane array be precise to  $10 \mu\text{m}$ , for the physical location of any corner of a CCD. This implies a typical rotation of the CCDs (with respect to their nominal orientation) of  $10 \mu\text{m}/60 \text{mm} = 10 \mu\text{as}/60 \text{mas} = 30 \mu\text{as}$  per AC pixel. Thus, if the AC coordinate would be known to 1 AC pixel only, AL transformation errors of several dozen microarcsec would result. Effects of similar size must be expected from optical distortion, image rotation etc. (a different assumption for the glueing precision would thus not help much).

An AF-external knowledge of the AC position of all objects to about 1 mas is thus desirable in order to ensure correctness of the transformation to the order of  $1 \mu\text{as}$ . It is clear that this will easily be achieved in the later stages of the Gaia mission and data reduction. For the majority of the faint stars it will not be available in the early stages, neither from the SM transits, nor from a star catalogue. However, even for faint stars the SM transits will be sufficient from the very beginning, since the precision of their individual AF measurements is lower, too.

TABLE 1: A possible specification of input data for object processing based on astrometric elementary observations (e.g. for an astrometric binary). Multiplicity is the total number of times the data item appears for a given object. The flags could code the FOV index and CCDs used in each FOV transit, as well as some quality or reliability information for the observations. Single-precision reals (float) are used whenever there is no risk of compromising accuracy at the  $1 \mu\text{as}$  level. Assuming  $N = 80$ , the total amount of data is about 4 kByte per object, or 4 TB for all  $10^9$  objects.

Notes: 1) No photometric data are included. 2) The reference points may possibly be omitted, and taken from the astrometric source catalogue instead. 3) This concept is not applicable to solar-system objects. 4) The data set would be increased by about a factor of 8–9 if some data ( $w, z, \sigma_w, \sigma_z$ ) would be given for each CCD transit individually, instead of averages over one field-of-view transit. In this case, information on the correlation between the different CCD transits would have to added, too (probably just one quantity per field-of-view transit).

Quantity	Designation	Type	Multiplicity	Bytes
Data given once per object:				
identifier	–	long	1	8
reference point in RA	$\alpha_0$	double	1	8
reference point in Dec	$\delta_0$	double	1	8
number of FOV transits	$N$	int	1	4
Data given once per FOV transit:				
time	$t$	long	$N$	$8N$
flags	–	int	$N$	$4N$
position angle of scan	$\theta$	double	$N$	$8N$
parallax factor AL	$f_w$	float	$N$	$4N$
parallax factor AC	$f_z$	float	$N$	$4N$
local coordinate AL of image centroid	$w$	double	$N$	$8N$
local coordinate AC of image centroid	$z$	float	$N$	$4N$
standard error AL	$\sigma_w$	float	$N$	$4N$
standard error AC	$\sigma_z$	float	$N$	$4N$

Total size =  $48N + 28$  bytes.

TABLE 2: A possible specification of input data for object processing and 2-d imaging based on samples in rectangular windows (e.g., for a partially resolved binary). The flags could code the FOV index and CCDs used in each transit, as well as some quality or reliability information for the observations. Assuming  $N = 80$ ,  $M = 10$  and window size  $(I, L) = (6, 1)$  (faint stars), the total amount of data is about 45 kByte per object, or 45 TB for all  $10^9$  objects.

Notes: 1)–3) as for Table 1. 4) The sample sizes and shear terms may significantly change over a field-of-view transit. Therefore they are given for each CCD transit separately. 5) The position angle of scan is given only once per field-of-view transit. It is a formal quantity relating the local scan coordinates to the ICRS. Small changes of the actual position angle of scan are of no relevance therefore. Still, in order to avoid any systematic effects,  $\theta$  should be chosen as the actual position angle of scan at the mid-time of a field-of-view transit, i.e. at the readout time for AF4. Similarly, the parallax factors are given only once, since they remain constant over a minute of time, even for the closest stars.

Quantity	Designation	Type	Multiplicity	Bytes
Data given once per object:				
identifier	–	long	1	8
reference point in RA	$\alpha_0$	double	1	8
reference point in Dec	$\delta_0$	double	1	8
number of FOV transits	$N$	int	1	4
Data given once per FOV transit:				
time	$t$	long	$N$	$8N$
flags	–	int	$N$	$4N$
position angle of scan	$\theta$	double	$N$	$8N$
parallax factor AL	$f_w$	float	$N$	$4N$
parallax factor AC	$f_z$	float	$N$	$4N$
number of samples AL	$I$	int	$N$	$4N$
number of samples AC	$L$	int	$N$	$4N$
number of CCD transits	$M$	int	$N$	$4N$
Data given once per CCD transit:				
local coordinate AL of sample (0,0)	$w_0$	double	$NM$	$8NM$
local coordinate AC of sample (0,0)	$z_0$	float	$NM$	$4NM$
sample size AL	$\Delta w$	float	$NM$	$4NM$
sample size AC	$\Delta z$	float	$NM$	$4NM$
shear term AL	$c_w$	float	$NM$	$4NM$
shear term AC	$v_z$	float	$NM$	$4NM$
sample values	$S_{it}$	float	$NMIL$	$4NMIL$

Total size =  $4NMIL + 28NM + 40N + 28$  bytes.

TABLE 3: A possible specification of input data for object processing and 2-d imaging containing both astrometric elementary observations and window sample data. The flags could code the FOV index and CCDs used in each transit, as well as some quality or reliability information for the observations. Assuming  $N = 80$ ,  $M = 10$  and window size  $(I, L) = (6, 1)$  (faint stars), the total amount of data is about 46 kByte per object, or 46 TB for all  $10^9$  objects.

Quantity	Designation	Type	Multiplicity	Bytes
Data given once per object:				
identifier	–	long	1	8
reference point in RA	$\alpha_0$	double	1	8
reference point in Dec	$\delta_0$	double	1	8
number of FOV transits	$N$	int	1	4
Data given once per FOV transit:				
time	$t$	long	$N$	$8N$
flags	–	int	$N$	$4N$
position angle of scan	$\theta$	double	$N$	$8N$
parallax factor AL	$f_w$	float	$N$	$4N$
parallax factor AC	$f_z$	float	$N$	$4N$
local coordinate AL of image centroid	$w$	double	$N$	$8N$
local coordinate AC of image centroid	$z$	float	$N$	$4N$
standard error AL	$\sigma_w$	float	$N$	$4N$
standard error AC	$\sigma_z$	float	$N$	$4N$
number of samples AL	$I$	int	$N$	$4N$
number of samples AC	$L$	int	$N$	$4N$
number of CCD transits	$M$	int	$N$	$4N$
Data given once per CCD transit:				
local coordinate AL of sample (0,0)	$w_0$	double	$NM$	$8NM$
local coordinate AC of sample (0,0)	$z_0$	float	$NM$	$4NM$
sample size AL	$\Delta w$	float	$NM$	$4NM$
sample size AC	$\Delta z$	float	$NM$	$4NM$
shear term AL	$c_w$	float	$NM$	$4NM$
shear term AC	$v_z$	float	$NM$	$4NM$
sample values	$S_{il}$	float	$NMIL$	$4NMIL$

Total size =  $4NMIL + 28NM + 60N + 28$  bytes.