Extended Geometric Calibration Model for Gaia's Astro Instrument

U. Bastian
ARI Heidelberg

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Applicable documents: L. Lindegren, Algorithms for GDAAS Phase II: Definition, GAIA-LL-044, V. 4.
U. Bastian, Reference Systems, Conventions and Notations for Gaia, GAIA–ARI–BAS-003, Issue 4.0
L. Lindegren, Geometric Calibration Model for the 1M Astrometric GIS Demonstration, GAIA-C3-TN-LU-LL-063-1
H. Lenhardt, S. Jordan, Software structure of the Gaia ODIS, GAIA-ARI-HL-003

Summary:

Three useful extensions to the Astro geometric calibration model of GAIA-LL-044 and GAIA-C3-TN-LU-LL-063-1 are proposed. Their implementation in the ESAC AGIS towards the end of 2006 is aimed at. They will definitely be needed in the operational Gaia system, both for the AGIS and for the First Look processing.

There is a consequence for the OBDH (on-board data handling), discussed in Section 8.

The present issue 2 corrects a mistake concerning the orthogonality conditions and makes a few updates of the text towards the Gaia-3 instrument configuration.

1 Introduction

In GAIA-LL-044, Section 1.6 and Appendix A, L. Lindegren defines the model for the geometric calibration of the Astro instrument which is used in GDAAS-2. In GAIA-C3-TN-LU-LL-063-1 this is summarized in a more compact form, to be used during 2006 for the simulations and the ESAC AGIS.

The geometric model calibration parameters are separated into "large-scale" and "small-scale" ones. There is one large-scale parameter $\Delta\eta$ and one $\Delta\zeta$ for each CCD, separately for each FoV (field of view), i.e. 138 parameters[1] in total for each field coordinate ($\Delta\eta$ and $\Delta\zeta$). These parameters describe a simple shift of the entire CCD with respect to the zero point of the respective field coordinate. The small-scale parameters $\delta\eta, \delta\zeta$ give analogous shifts, but with one parameter per pixel column, and with identical values in the two FoVs. So there are $76 \times 1966 = 149\,416$ small-scale parameters for each field coordinate. The philosophy behind this scheme is explained in Appendix A of GAIA-LL-044 and summarized again in GAIA-C3-TN-LU-LL-063-1. The large-scale and small-scale parameters are expected to be determined in independent time intervals, those for the small-scale parameters supposedly being much longer. The above-mentioned numbers of parameters are to be understood as the number per time interval. Orthogonality between large-scale and small-scale parameters is guaranteed by suitable orthogonality conditions.

[1] there are $7\times(2+9)-1=76$ CCDs, but for SM 1 and SM 2 there is only one FoV each

2 CCD Rotation, Optical Distortion, Focal-Length Changes

The large-scale parameters of the GAIA-LL-044 model take care of CCD displacements caused by non-nominal glueing of the CCDs to the focal-plane assembly, non-nominal basic angle, non-nominal focal length etc. But they do not take care of rotations of the CCDs caused by the glueing, by optical field rotation and by optical distortions. They also do not take care of the "stretching" of the CCDs due to a non-nominal focal length, non-nominal temperature etc., nor of the "bending" of the CCDs due to optical distortion.

All these effects have to be taken care of by the small-scale parameters. Now, this has several important disadvantages:

i) To calibrate the hundreds of thousands of small-scale parameters to sufficient precision necessitates a very large number of measurements, i.e. quite a long time interval. This will give problems during commissioning and initial calibration, as well as in the First Look processing.

ii) Contrary to those small-scale irregularities that are really due to CCD inhomogeneities, the effects listed above (most notably image rotation and optical distortion) need not be identical in the two FoVs, and surely will not be.

iii) In addition, they will not necessarily be sufficiently constant over long time intervals, thus possibly reducing the precision of the global astrometric solution.

iv) Any change in the optical distortion or image rotation will lead to unnecessarily large residues in First Look, thus reducing First Look's ability to diagnose the inherent precision of the measurements and the geometric stability. Daily calibration of the large number of small-scale parameters would, on the other hand, grossly reduce the overall precision of the calibration.

It should be noted that the effects of rotation, stretching and distortion will be of the same order of magnitude as the shifts, and have similar time scales for variations.

I therefore propose to add more large-scale parameters, in the form of orthogonal polynomials in the across-scan coordinate for each CCD and FoV. The orthogonality conditions of GAIA-LL-044 then have to be extended slightly, see Section 5. The obvious choice for the polynomials are shifted Legendre polynomials $L_n^\star$ in a coordinate $\tilde{\mu}$ that runs from zero to 1 on the trailing across-scan edge of each CCD.

So, Equation (1) in Appendix A of GAIA-LL-044, viz.

$$\eta^{(obs)} = \eta^0 + \Delta\eta + \delta\eta \tag{1}$$

is changed into

$$\eta^{(obs)} = \eta^0 + \Delta\eta_0 L_0^\star(\tilde{\mu}) + \Delta\eta_1 L_1^\star(\tilde{\mu}) + \Delta\eta_2 L_2^\star(\tilde{\mu}) + \delta\eta \tag{2}$$

where η^0 is the nominal location of the CCD, $\Delta\eta_0, \Delta\eta_1, \Delta\eta_2$ are the newly defined calibration parameters, and $L_0^\star, L_1^\star, L_2^\star$ are the shifted Legendre polynomials

$$\begin{aligned} L_0^\star(\tilde{\mu}) &= 1 = const \\ L_1^\star(\tilde{\mu}) &= 2(\tilde{\mu} - \frac{1}{2}) \\ L_2^\star(\tilde{\mu}) &= 6(\tilde{\mu} - \frac{1}{2})^2 - \frac{1}{2} \end{aligned} \tag{3}$$

Because $L_0^\star$ is a constant, $\Delta\eta_0$ is exactly equal to $\Delta\eta$ of GAIA-LL-044. The normalization of the shifted Legendre polynomials is chosen such that the value $L_n^\star(1) = 1$ for all n. So they are not orthonormal in the usual sense. The normalization to $L_n^\star(1) = 1$ is better suited for our purposes than the orthonormal version, because the $\Delta\eta_n$ directly illustrate the maximum effect of each term.

If need be, more polynomials can be added of course. But their number should remain very small compared to 1966. It is expected that polynomials higher than $L_2^\star$, or at most $L_3^\star$, will not be necessary.

3 Across-scan Calibration for all CCD Strips

GAIA-LL-044 assumes that an across-scan calibration is performed for SM only. This should be changed, and this can indeed be changed.

First, across-scan measurements are performed not only on SM, but also on AF1 for all images.

Second, an across-scan calibration is needed even for those CCD strips where no across-scan measurements are performed, in order to make correct across-scan placement of the windows possible.

An across-scan calibration for all CCD strips of AF needs images that are resolved across scan, of course. In older versions of the Gaia sampling strategy, such images had not been foreseen for AF2 ff. Therefore an across-scan calibration was thought to be impossible. However, in newer versions of the sampling strategy there is a bright-star observation mode avoiding the strong on-chip binning of samples across scan. This mode provides the information necessary for the across-scan calibration in AF2–9.

Therefore, the calibration model given in the previous section will be extended by an across-scan analogue:

$$\zeta^{(obs)} = \zeta^0 + \Delta\zeta_0 L_0^\star(\tilde{\mu}) + \Delta\zeta_1 L_1^\star(\tilde{\mu}) + \Delta\zeta_2 L_2^\star(\tilde{\mu}) + \delta\zeta \tag{4}$$

It is, of course, clear that the across-scan calibration for AF2–9 is not needed to the same precision as that for the SM and AF1, and as that along scan.

4 Formulation in Pixel Coordinates

GAIA-LL-044 does not introduce a specific coordinate for the representation of across-scan locations. As explained in GAIA-ARI-BAS-003, Section 6, a geometric calibration ultimately means a transformation from pixel coordinates to field angles (or cartesian field coordinates, if more appropriate) — and vice versa.

Therefore, the normalized across-scan coordinate $\tilde{\mu}$ appearing in the equations of the present paper must be defined as

$$\tilde{\mu} = \mu/1966 \tag{5}$$

where μ is the CCD-specific across-scan pixel coordinate defined in GAIA-ARI-BAS-003, and 1966 is the total number of pixel columns of the CCD.

5 Orthogonality Conditions

The orthogonality conditions of GAIA-LL-044 have to be extended slightly, since now the small-scale parameters have to be orthogonal not only to the CCD shifts (the zero-order Legendre terms), but to all the Legendre terms independently. Furthermore, the orthogonality conditions for the large-scale parameters now have to explicitly refer to the zero-order polynomials. For ease of reference the complete set of conditions is given in the following.

The large-scale along-scan calibration parameters are subjected to this constraint:

$$\sum_{FoV=1}^{2} \sum_{j=strips} \sum_{k=rows} \Delta\eta_0(FoV, j, k) = 0 \tag{6}$$

with *strip* and *row* describing the location of an individual CCD within the focal plane, and FoV being the field-of-view index. This constraint effectively defines the zeropoint of the along-scan field coordinates, thus preventing a degeneracy between the along-scan calibration and the along-scan attitude in the global astrometric solution.

Analogously, in order to prevent a degeneracy between the across-scan calibration and the across-scan attitude, the zeropoint of the across-scan field coordinates must be defined. Since the across-scan attitude has two degrees of freedom, the across-scan large-scale parameters in the two fields of view must be constrained independently. Hence the following constraints are needed:

$$\sum_{j=strips} \sum_{k=rows} \Delta\zeta_0(FoV = 1, j, k) = 0 \tag{7}$$

$$\sum_{j=strips} \sum_{k=rows} \Delta\zeta_0(FoV = 2, j, k) = 0 \tag{8}$$

The small-scale parameters must be constrained to avoid a degeneracy with the large-scale parameters. Thus they must obey the constraints (*pixcols* stands for the total number of pixel columns, i.e. 1966):

$$\sum_{m=1}^{pixcols} \delta\eta_m L_n^\star(m/1966) = 0 \qquad \text{(separately for each CCD and for each Legendre order } n\text{)} \tag{9}$$

for the along-scan calibration, and similarly

$$\sum_{m=1}^{pixcols} \delta\zeta_m L_n^\star(m/1966) = 0 \qquad \text{(separately for each CCD and for each Legendre order } n\text{)} \tag{10}$$

for the across-scan calibration.

For naming conventions see GAIA-ARI-BAS-003, chapters 4 and 5.

6 Practical Computation of the Legendre Coefficients

This section is intended to remark — in a somewhat loose formulation, without going into the full details — that the practical computation of the Legendre coefficients is very simple.

The observation equations for the large-scale calibration unknowns read

$$(\eta_{obs} - \eta_{calc})_i = -\sum_n \Delta\eta_n L_n^\star(\tilde{\mu}_i) \tag{11}$$

where i enumerates the relevant observations for the calibration unit (meaning a combination of a CCD and a time interval, see GAIA-LL-044) under consideration, and n is the order of the shifted Legendre polynomials. The minus sign on the right-hand side is due to a peculiarity of the Gaia calibration adjustment task, viz. that the unknowns appear in the "observed" quantity η_{obs} rather than in the "calculated" quantity η_{calc}, as would be the usual case in least-squares problems.

The coefficient matrix of the adjustment problem thus reads

$$\frac{\partial(\eta_{obs} - \eta_{calc})_i}{\partial\Delta\eta_n} = -L_n^\star(\tilde{\mu}_i) \tag{12}$$

and the elements of the normal-equation matrix N correspondingly are

$$N_{nm} = \sum_i P_i L_n^\star(\tilde{\mu}_i) L_m^\star(\tilde{\mu}_i) \tag{13}$$

where P_i is the weight of the ith observation equation.

By construction of the shifted Legendre polynomials and by the uniform (random) distribution of the measurements i over the across-scan coordinate $\tilde{\mu}$ we may assume that the off-diagonal elements of N are negligible. Thus the inversion of N becomes trivial, and the solution of the normal equations simply reads as

$$\Delta\eta_n = \frac{-\sum_i P_i L_n^\star(\tilde{\mu}_i)(\eta_{obs} - \eta_{calc})_i}{\sum_i P_i \left[L_n^\star(\tilde{\mu}_i)\right]^2} \tag{14}$$

and analogously for the across-scan unknowns $\Delta\zeta_n$, using observations $(\zeta_{obs} - \zeta_{calc})_i$.

7 A Long-Term Comment on the Small-Scale Parameters

The goal of the small-scale parameters is to represent the genuine CCD irregularities (thus they are assumed to be identical in the two fields of view). However, the model implied by the 1966 parameters valid for 1966 disjoint intervals of the across-scan coordinate is not optimal. On the long run, but not within the next two or three prototype stages of the Gaia data reduction and simulations, a modification should be considered.[2]

First, the location of the images on the CCD are not determined by a discrete integer number, but by a continuous coordinate. Thus the true character of the CCD irregularities will not be a discontinuous function in the $\tilde{\mu}$ coordinate.

Second, the number of 1966 parameters per CCD is excessive. The across-scan extent of images is larger than one pixel. Therefore there is a correlation of the effect of the CCD irregularities over a few pixels across scan. This might be taken into account in order to reduce the number of unknowns.

8 A Requirement for the OBDH

The magnitude limit for the bright-star sampling mode for AF2–9 (using samples without on-chip binning across scan) must be switchable. During commissioning and initial calibration a lower-than-usual magnitude limit is needed in order to get a sufficient number of across-scan measurements within a reasonable time.

The small number of across-scan resolved images during nominal operations will then suffice to keep an eye on any slow evolution of the across-scan calibration, to the precision needed for window placement.

Should the bright-star sampling mode (as such) be dropped from future sampling strategies, then an unbinned observation mode for AF2–9 must still be implemented, for the sake of commissioning and initial calibration. It should be possible to temporarily switch it on in later phases of the mission in order to occasionally check the across-scan calibration for the sake of correct window placement.

[2] For the time being, the algebraic simplicity and the trivial orthogonality of the present model is most helpful